

It's all Connected:

Knowledge Representation for Graph Data



Informatics



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Graph Data

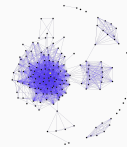
Graphs are everywhere



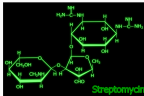
Transport networks



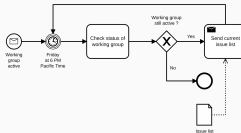
Interactome



Social networks



Molecules



Business processes



Flow charts

Fundamental abstractions for **modelling** and **computation**.

Graphs as a Data Model

Graph data is increasingly adopted as a **data model**, particularly when **connections between nodes** convey important information.

Papers affected by the retraction of AB24 to degree ≤ 3

Graph query: $AB24 \xrightarrow{\text{cites}^- \cdot (\varepsilon | \text{cites}^-) \cdot (\varepsilon | \text{cites}^-)} x$ **vs. SQL:**

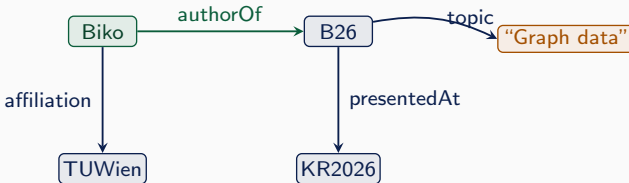
```
SELECT src FROM Cites WHERE dst='AB24'
UNION
SELECT c2.src FROM Cites c1 JOIN Cites c2
ON c1.src=c2.dst WHERE c1.dst='AB24'
UNION
SELECT c3.src FROM Cites c1
JOIN Cites c2 ON c1.src=c2.dst
JOIN Cites c3 ON c2.src=c3.dst
WHERE c1.dst='AB24'
```

Two **graphs-as-data** paradigms: RDF and Property Graphs

Resource Description Framework (RDF) triple stores

RDF: open standard for sharing data on the web

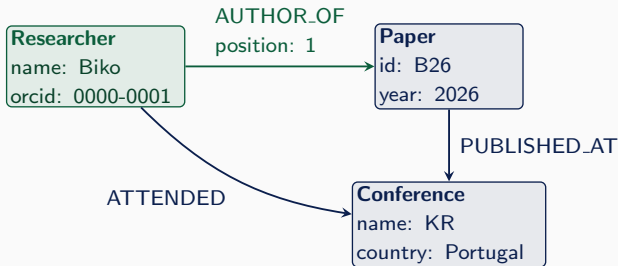
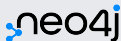
- format of knowledge graphs and open data repositories
- labelled graphs as **triples** (s, p, o)
 - (Biko, authorOf, B26) (Biko, affiliation, TUWien)
 - (B26, presentedAt, KR2026) (B26, topic, "Graph data")
- SPARQL query language, access via (web) endpoints



Property Graphs

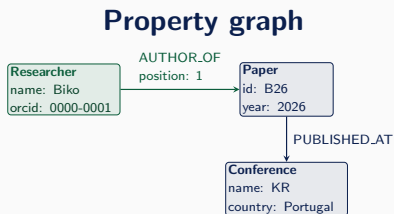
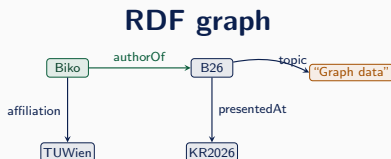
Property graphs (PGs): data model of graph databases

- Labelled graph with property values in nodes and edges
- Broad industrial adoption
- A zoo of proprietary engines and query languages



New standards for Graph Data

Different models with **a large common core**



Recent standardisation efforts

aim at narrowing the gap.

Description Logics: KR for Graphs

Description Logics (DLs) are logics for describing graphs



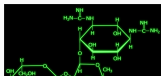
Tram $\sqsubseteq$
 $\forall \text{stopsAt.Station}$



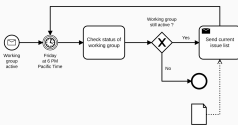
Complex $\sqsubseteq$
 $\exists \text{hasPart.Protein}$



Researcher $\sqsubseteq$
 $\exists \text{collaborates.}$
Researcher



Macrolide $\sqsubseteq$
 $\exists \text{hasPart.}$
LactoneRing



Approved $\sqsubseteq$
 $\exists \text{hasStep.Review}$



Decision $\sqsubseteq$
 $\forall \text{next.}$
(Decision $\sqcup$ Action)

DLs: a flexible toolbox of decidable logics

	NL	P	ExpTime	
	DL-Lite	$\mathcal{EL}$	$\mathcal{ALCI}$	$\mathcal{SHIQ}$
$A \sqsubseteq B$	✓	✓	✓	✓
$A \sqsubseteq B \sqcup C$			✓	✓
$A \sqsubseteq \exists r.B$	✓	✓	✓	✓
$A \sqcap \exists r.B \sqsubseteq C$		✓	✓	✓

- well-charted complexity vs. expressiveness trade-off
- highly optimised off-the-shelf reasoners for many DLs
- plethora of services: explanations, repairs, updates, refinements, forgetting, module extraction, ...



DLs have been a powerful KR tool in data management:

- To describe background knowledge and infer implicit facts
- As constraint languages describing requirements on datasets
- As query languages
- As foundations for ontology-mediated data access

Today's Talk: DLs are ready to face new challenges!

Part 1: revisiting an old problem

Can we finally have true ontology-mediated querying?

Part 2: addressing new ones

KR as foundations for new graph constraint languages

Part 1: revisiting an old problem

Ontology-Mediated Query Answering

An ontology-mediated query (OMQ)

is a pair $(q, \mathcal{O})$: a database query q together with a DL ontology $\mathcal{O}$ providing background knowledge.

OMQ $(q, \mathcal{O})$

$q(x) = \text{Paper}(x)$

$\exists \text{ authored}^{\perp}.T \sqsubseteq \text{Paper}$
 $\exists \text{ publishedIn}.T \sqsubseteq \text{Paper}$

Data $\mathcal{D}$

$\text{authored}(\text{An}, \text{ANV26})$
 $\text{authored}(\text{Biko}, \text{ABK23})$
 $\text{publishedIn}(\text{KR24}, \text{AIJ})$

$\vdots$

$$\text{Ans}_{\mathcal{O}}(q, \mathcal{D}) = \{\text{ANV26}, \text{ABK23}, \text{KR24}\}$$

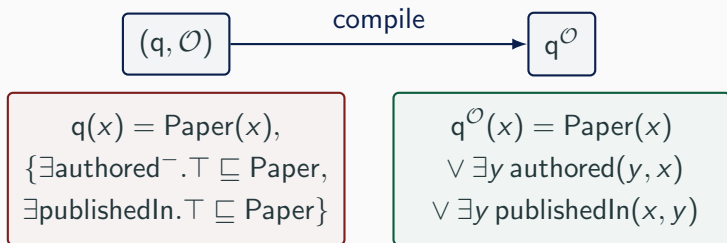
- infer missing facts from incomplete data
- define high-level vocabulary for better query formulation
- unified access to heterogenous data ...

Query Rewriting Makes OBDA Possible

OMQs are the pillar of **virtual knowledge graphs (VKGs)**, one of the biggest DL success stories of the last decade.

OMQ rewriting

Compile $(q, \mathcal{O})$ into a *pure* database query $q^{\mathcal{O}}$ that gives the same answers over every database.



Key: $q^{\mathcal{O}}$ can be evaluated with an ordinary database engine

For two decades, we have been stuck with un-graphy OMQs

Relational-like queries

joins, but no graph navigation:

$$r(x, y) \wedge r(x, z) \wedge s(z, y).$$

No reachability $r^*(x, y)$.

DL-Lite ontologies

Deliberately weak ontology language:

$$B \sqsubseteq C \quad B \sqsubseteq \exists r \quad \exists r \sqsubseteq B$$

Local, blind to graph structure.

Every graph query language has some form of **graph pattern matching** using **path expressions**

Papers affected by a retraction $q(x) = \langle \text{cites}^+ \cdot \text{Retrd?} \rangle?(x, y)$

- A **2-way regular path query (2RPQ)** is an atom $E(x, y)$ with E a regular expression over (possibly inverted) edge labels.
- Conjunctive 2RPQs join 2RPQs through shared variables

$$q(\vec{x}) = \exists \vec{y} \bigwedge E_i(u_i, v_i)$$

Navigational OMQs are not rewritable into C2RPQs

$$q(x, y) = (r \cdot A?)^+(x, y), \quad \mathcal{O} = \{\exists s. \top \sqsubseteq A\}.$$

A regular path cannot leave the path to check s -edge and return.

Nested paths let us branch off tests without leaving the main path.

$$q^{\mathcal{O}}(x, y) = (r \cdot (A? + \langle s \rangle?))^+(x, y).$$

- OMQs with *nested regular expressions* studied a decade ago
- but graph database engines did not support them!

The barrier is finally overcome!

The New Standards Supply the Missing Expressiveness

GQL: new standard for querying PGs (ISO/IEC 39075:2024)

- commercial engines are quickly moving towards GQL support
- SQL/PGQ (SQL:2023) defines graph-pattern queries over property-graph views of relational data

Their graph-pattern core supports nested path navigation

- the target language needed for DL-Lite rewritings now exists!
- Ten-year-old techniques are finally implementable.
- Ongoing work on making the techniques more goal-oriented and the rewritings more succinct.
- First prototype compiling OWL QL + CN2RPQ into Cypher.

What About Going Beyond DL-Lite?

DL-Lite is too poor for describing graphs, e.g., it cannot propagate labels to/from neighbouring nodes.

$$\exists r.\text{Red} \sqsubseteq \text{Blue} \quad \text{Blue} \sqsubseteq \forall r.\text{Red}$$

- DL-Lite reasoning is in AC_0 .
- Graph query evaluation is NL-complete in data complexity.

Can we spend the additional complexity available in graph query languages on a more expressive ontology language?

Unfortunately, the answer is not in the DL toolbox

$\mathcal{EL}$: conjunction simulates monotone circuits

$\text{And} \sqcap \exists \text{left.True} \sqcap \exists \text{right.True} \sqsubseteq \text{True},$

$\text{Or} \sqcap \exists \text{left.True} \sqsubseteq \text{True},$

$\text{Or} \sqcap \exists \text{right.True} \sqsubseteq \text{True}.$

$\mathcal{ELI}^{lin}$: inferences recover conjunction

$$A \sqcap B \sqsubseteq C \rightsquigarrow \left\{ \begin{array}{l} A \sqsubseteq \exists r.T, \\ \exists r^-.B \sqsubseteq D, \\ \exists r.D \sqsubseteq C. \end{array} \right.$$

Entailment is P-hard in data complexity.

Can We Recover a Rewritable Fragment?

$\mathcal{ELHI}$ provides the graph features that DL-Lite lacks:

- conjunction of node labels,
- existential restrictions on either side of inclusions,
- inverse roles and role inclusions.

Keep these constructors, but stratify their recursive interaction so that ontology reasoning remains navigational.

$\mathcal{ELHI}_{\preceq}$

Stratifying Conjunctions

The fragment $\mathcal{ELHI}_{\preceq}$

An ontology $\mathcal{O}$ belongs to $\mathcal{ELHI}_{\preceq}$ if its axioms have the form

$$A \sqsubseteq B, \quad A \sqcap B \sqsubseteq C, \quad A \sqsubseteq \exists s.B, \quad \exists s.A \sqsubseteq B, \quad r \sqsubseteq s.$$

Stratification is witnessed by a preorder $\preceq$:

- predicates on the left are no larger than the predicate on the right;
- the comparison is strict for all but at most one left-hand predicate;
- inverse roles occupy the same stratum.

Recursion is allowed; recursive conjunction is controlled.

Stratification Brings Reasoning Back into NL

Query rewriting

For every $\mathcal{ELHI}_{\preceq}$ -ontology $\mathcal{O}$ and N2RPQ q , there is an UCN2RPQ q' such that

$$(\mathcal{D}, \mathcal{O}) \models q(a) \iff \mathcal{D} \models q'(a).$$

- A nested automaton simulates derivations with bounded alternation.
- Each atomic query compiles into a nested path query.

For $\mathcal{ELHI}_{\preceq}$, CN2RPQs rewrite into equivalent CN2RPQs and can be evaluated directly over the graph data.

The Fragment Covers Most Tested $\mathcal{ELHI}$ Ontologies

Preliminary check on the ORE 2015 corpus:

455/509 = 89.4% of the $\mathcal{ELHI}$ ontologies admit an $\mathcal{ELHI}_{\preceq}$ stratification.

Natural recursive propagation remains available

$\exists \text{partOf}.\text{Contaminated} \sqsubseteq \text{Contaminated}.$

Mutually recursive conjunction is excluded

$\exists \text{contacted}.\text{Infected} \sqcap \exists \text{coResident}.\text{Infected} \sqsubseteq \text{Infected}.$

We can finally have ontology-mediated graph querying.

Part 2: addressing new ones

SHACL: A Short History

- Web data stores on the web are enormous
- They are interconnected and usually open
- RDF is schema-less: data can be heterogeneous and messy

(P1, author, An) (An, authorOf, P1) (P1, dc:creator, An)

We need a way to **describe and validate** datasets!

- W3C RDF Data Shapes Working (2014): let's standardize a language for structural constraints on RDF graphs!
- The Shapes Constraint language **SHACL** becomes a **W3C Recommendation in 2017**.

Shape expressions (Logic syntax)

$$\varphi ::= c \mid A \mid \varphi \mid \neg\varphi \mid \varphi \wedge \varphi \mid \exists_{\geq n} E.\varphi \mid \text{eq}(E, r) \mid \text{disj}(E, r)$$

where c is a node name (constant),
 A is a class name (unary predicate),
 r is a relation name (binary predicate), and
 E is a **regular expression** over roles and their inverses.

$\varphi \vee \varphi$, $\exists E.\varphi$, and $\forall E.\varphi$ expressible as usual

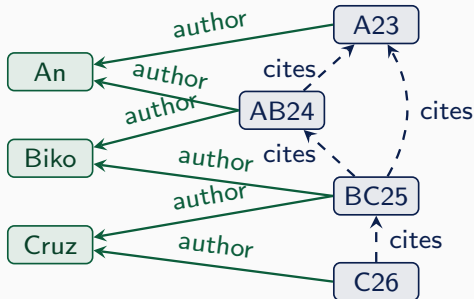
Shapes are **concepts** in a very **expressive description logic**

Semantics of SHACL Shapes: an example

Semantics as expected

$\llbracket \varphi \rrbracket^G$ denotes the **set of nodes** in G that are in the extension of φ

$$\varphi = \exists_{\geq 2} \text{author}$$



$$\llbracket \varphi \rrbracket^G = \{AB24, BC25\}$$

Shape Validation

given a graph G , a **target node** n , and a shape φ ,
decide whether n **validates** φ in G $n \in \llbracket \varphi \rrbracket^G$



$\varphi = \exists \text{authorOf.Paper}$
Target: Biko
 $\text{Biko} \in \llbracket \varphi \rrbracket^G \checkmark$

Validation of a shape is **model checking** over a **given graph**

- No open-world, no completion
- An **interpretation**, not an ABox

1. Catalogue of shape definitions

$$S_1 \equiv \varphi_1 \quad \cdots \quad S_n \equiv \varphi_n$$

- defined shape names $S_1, \dots, S_n$
- each S_i has a definition φ_i
- shape expressions φ can mention **shape names** $S_1, \dots, S_n$

$\text{PAINTER} \equiv \exists \text{creatorOf}.\text{PAINTING}$

$\text{PAINTING} \equiv \exists \text{creatorOf}^{\neg}.\text{PAINTER}$

2. **Targets** pairs (n, S) for which we want to validate $n \in \llbracket S \rrbracket^G$

- more complex targets allowed
e.g. *class targets*: validate S for all objects in class C ,

From Model Checking to Model Finding

$$S_1 \equiv \varphi_1 \quad \cdots \quad S_n \equiv \varphi_n,$$

Solution (aka shape assignment, model)

node labellings S that satisfy all shape definitions: $\llbracket S_i \rrbracket_S^G = \llbracket \varphi_i \rrbracket_S^G$

$\text{PAINTER} \equiv \exists \text{creatorOf}.\text{PAINTING}$

$\text{PAINTING} \equiv \exists \text{creatorOf}^-. \text{PAINTER}$



Solutions:

$$\llbracket \text{PAINTER} \rrbracket_{S_1}^G = \{\}$$

$$\llbracket \text{PAINTING} \rrbracket_{S_1}^G = \{\}$$

$$\llbracket \text{PAINTER} \rrbracket_{S_2}^G = \{\text{Biko}\}$$

$$\llbracket \text{PAINTING} \rrbracket_{S_2}^G = \{\text{KR25}\}$$

Semantics of Recursive SHACL

Not all models are equally intuitive. . .

moreover, **validation becomes intractable!**

Maybe we need some form of minimality?

Positive SHACL

- If the defined shapes occur positively, there's a unique minimal model derivable by a fixed point computation

Close to monadic regular Datalog and modal substitution calculus.

But in general, no unique obvious semantics

$\text{PAINTER} \equiv \text{Artist} \wedge \neg \text{WRITER}$ $\text{WRITER} \equiv \text{Artist} \wedge \neg \text{PAINTER}$

Recursion in the Specification

According to <https://www.w3.org/TR/shacl/>:

W3C Recommendation

A shape **s1** in an RDF graph **G** **refers** to shape **s2** in **G** if it has **s2** as [value](#) for some non-list-taking, shape-expecting parameter of some constraint component or **s2** as a [member](#) of the [value](#) for some list-taking, shape-expecting parameter of some constraint component. A shape in an RDF graph **G** is a **recursive shape** in **G** if it is related to itself by the transitive closure of the [refers](#) relationship in **G**.

The [validation](#) with [recursive](#) shapes is not defined in SHACL and is left to SHACL processor implementations. For example, SHACL processors may support recursion scenarios or produce a failure when they detect recursion.

The expectation is that future work . . . will lead to the definition of specific dialects of SHACL where recursion is well-defined.

Stratified SHACL

- **Stratified negation** only over atoms defined in lower strata.
- One natural **perfect** model computable in **polynomial time**.

Full recursive SHACL: three choices

- **Supported**: accept all solutions. **Intractable!**
- **Stable**: well-known from Logic Programming.
Intuitively, only choices where everything is justified.
Hard to compute, but tractable for stratified definitions.
- **Well-founded**: unique three-valued model.
Polynomial approximation of the stable semantics.
Well-suited for SHACL validation.

With the KR toolbox, we have managed to address a few SHACL challenges:

- Validation of incomplete graphs: combining SHACL+ontologies
- Static analysis: SHACL satisfiability and implication
- Validation preservation in evolving graphs
- Learning SHACL from data

Validating SHACL in the Presence of Ontologies

- **SHACL**: closed-world validation
- **Ontologies**: implicit facts, open-world view.

Goal: validate incomplete graphs with ontology consequences.



$Biko \in \llbracket \text{AUTHOR} \rrbracket^{\text{cl}_O(G)}$

Ontology axiom

$\text{firstAuthorOf} \sqsubseteq \text{authorOf}$

Shape and target

$\text{AUTHOR} \equiv \exists \text{authorOf}.\top$

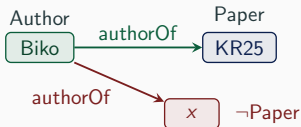
target: Biko

From Universal to Core Models

Data graph



Skolem chase



Austere model



Ontology axiom

$\text{Author} \sqsubseteq \exists \text{authorOf}.\top$

Shape

$\text{ONLYPAPERS} \equiv$
 $\forall \text{authorOf}.\text{Paper}$

$\text{Biko} \notin \llbracket \text{ONLYPAPERS} \rrbracket^{G_{\text{sk}}} \quad \times$

$\text{Biko} \in \llbracket \text{ONLYPAPERS} \rrbracket^G \quad \checkmark$

We need a core that adds only genuinely needed successors.

Ontology-Aware Validation: Contributions

- Semantics based on validation over **austere canonical model**
 - prove that it is a **core**
 - construction does not require repeated core tests
- Rewriting of SHACL plus an $\mathcal{ELHI}$ TBox into plain SHACL
 - ontology consequences compiled into SHACL constraints
 - output checkable by ordinary validators
- Validation is ExpTime-complete combined complexity.
- Validation is PTime-complete data complexity.
- Preliminary ideas towards a practical rewriting: data-aware rewriting builds a compact core representation

SHACL Satisfiability and Implication

Static analysis questions

- Is a SHACL shape satisfied in some graph?
- Are two SHACL shapes equivalent?
That is, they have the same extension on every graph
- Does one shape imply another?
- Over arbitrary and over finite graphs

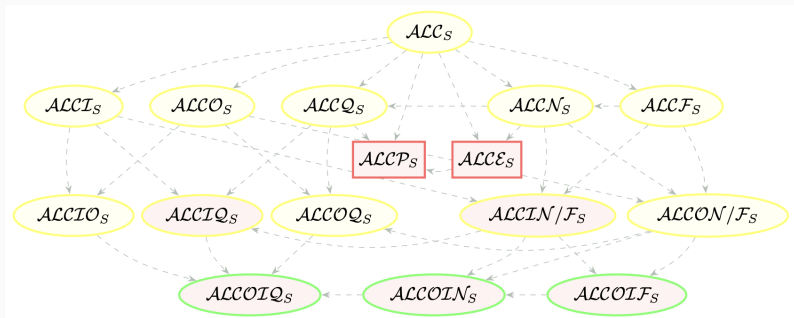
Transfer results from the DL literature! (Oudshoorn 2025)

- Many core fragments inherit ExpTime bounds from DLs.
- Richer counting fragments reach NExpTime or undecidability.
- Finite satisfiability is more fragile: several fragments lack the finite model property.

SHACL Satisfiability: Complexity Landscape

Using DL syntax:

(Oudshoorn 2025)



Ellipses: decidable; squares: undecidable.

Border color distinguishes ExpTime from NExpTime.

Yellow fill indicates means finite model property.

Static Analysis of SHACL Documents

Document satisfiability and implication (definitions+targets)

Given documents (C_1, T_1) and (C_2, T_2) , decide whether every graph validating (C_1, T_1) also validates (C_2, T_2) .

Main results

- For supported and stable semantics: **undecidable**
 - stratification regains decidability in the stable case
 - but not in the supported case
- For the well-founded semantics: **ExpTime-complete**.

Upper bound

Translation of well-founded SHACL into full hybrid μ -calculus

Don't miss Anouk's talk tomorrow 11:25! (B1.03)

SHACL Validation Under Graph Updates

Static validation of RDF graph updates

Given a (sequential) update $\mathcal{U}$ and a SHACL specification $\mathcal{C}$, does **every graph** that **satisfying** $\mathcal{C}$ still satisfy it **after applying** $\mathcal{U}$?

- RDF graphs evolve; avoid failure after updates.
- The paper defines a SHACL-based update language.
- Adds or removes labels based on shapes.

Main technique: regression

- update effects are compiled into SHACL constraints $tr^{\mathcal{U}}(\mathcal{C})$.
- Static validation reduces to satisfiability/containment.
- Undecidable in general, decidable fragments identified.

Considered setting

Fitting SHACL shapes from positive and negative example nodes in a graph, possibly using shape names from a recursive catalogue.

- Find a shape in a core SHACL fragment (essentially $\mathcal{ELI}^*$).
- All semantics considered: well-founded, stable, and supported.
- Fitting existence is ExpTime-complete for recursive SHACL under all three semantics.
- Most-specific fitting existence is in ExpTime.
- When validation is tractable, polynomial for bounded positive examples.

That's it for today

- Many open problems remain in the SHACL world
- Bridging the gap with other forms of graph reasoning and learning offers exciting perspectives!

Reliable reasoning from knowledge and data
still poses many challenges to KR.

- As new formalisms emerge, we face new challenges
- and reencounter old challenges in new forms,
- but we also find new opportunities to overcome previous limitations.