Rigorous Explainability by Feature Attribution

From SHAP to nuSHAP

Joao Marques-Silva

ICREA & Univ. Lleida, Spain

KR @ FLoC'26, July 2026

> *"All truths are easy to understand once they are discovered; the point is to discover them."*
>
> (G. Galilei)

ADVANCES OF AI IN RECENT YEARS: A TRANSFORMATIVE DECADE

SOURCE: INDUSTRY ANALYSIS AND RESEARCH (2024)

But news of human-level intelligence are greatly exaggerated...

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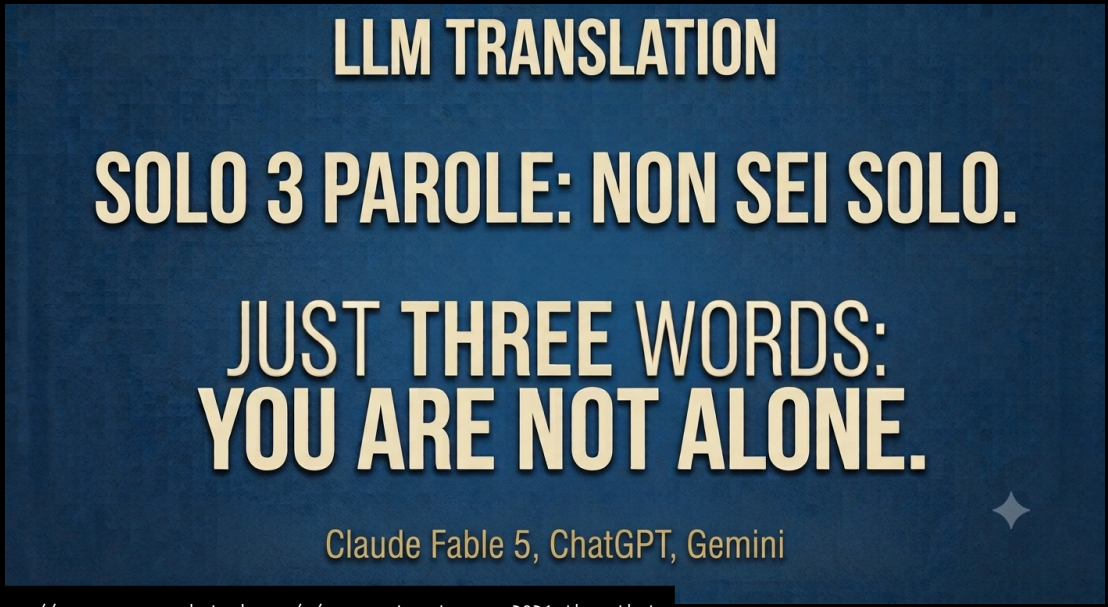


https://garymarcus.substack.com/p/you-cant-get-more-2026-than-that

Also, hallucinations are a critical weakness...

AI Hallucinations

When AI agents produce erroneous or misleading historical information

When did Angola become an independent country?

AI Agent

AI Response (Incorrect)

Angola has been an independent country since **1962.**

This is incorrect.
The independence date is wrong.

Historical Fact (Correct)

Angola has been an independent country since **11 November 1975.**

11 November 1975
Angola gained independence from Portugal.

Key Takeaway: AI systems can confidently generate incorrect historical information. **Always verify historical facts with reliable sources.**

Some clues of LLM hallucinations in recent papers...

VERGE: Formal Refinement and Guidance Engine for Verifiable LLM Reasoning

Wrong titles, authors, author list...

27 Jan 2026

Anton Belov, Mikoláš Janota, Inês Lynce, and Joao Marques-Silva. 2012. On computing minimal correction subsets. In *International Joint Conference on Artificial Intelligence*, pages 615–622.

Mark H Liffiton and Ammar Malik. 2008. Algorithms for computing minimal unsatisfiable subsets of constraints. *Journal of Automated Reasoning*, 40(1):1–33.

Some clues of LLM hallucinations in recent papers...

Concept-Based Abductive and Contrastive Explanations for Behaviors of Vision Models

Wrong titles, authors, author list...

7 May 2026

Alexey Ignatiev, Nina Narodytska, and João Marques-Silva. Formal explanations of neural network predictions. In *Advances in Neural Information Processing Systems 32 (NeurIPS 2019)*, pp. 12344–12354, 2019b.

Alexey Ignatiev, Hadi Izza, João Marques-Silva, Nina Narodytska, and Mate Soos. Using MaxSAT for efficient explanations of tree ensembles. In *Proceedings of the Thirty-Sixth AAAI Conference on Artificial Intelligence (AAAI 2022)*, volume 36, pp. 3776–3784, 2022.

Hadi Izza and João Marques-Silva. On explaining random forests with SAT. In *Proceedings of the Thirtieth International Joint Conference on Artificial Intelligence (IJCAI 2021)*, pp. 2584–2590, 2021.

THE IMPORTANCE OF EXPLAINABLE AI (XAI) FOR BUILDING TRUST

KEY BENEFITS OF XAI FOR TRUST

1. ENHANCED ACCOUNTABILITY
CLEAR RESPONSIBILITY FOR DECISIONS

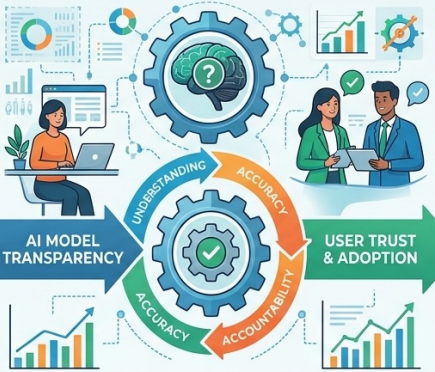
2. IMPROVED FAIRNESS & EQUITY
DETECTING AND MITIGATING BIAS

3. REGULATORY COMPLIANCE
MEETING ETHICAL & LEGAL REQUIREMENTS

4. USER UNDERSTANDING
MAKING AI REASONING AUDITABLE

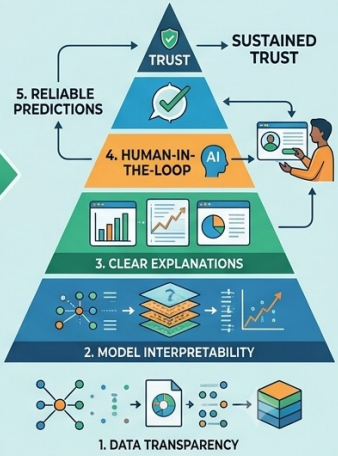
5. SYSTEM DEBUGGING & RELIABILITY & EDGE CASES

IDENTIFYING ERRORS & EDGE CASES



HOW XAI COUNTERS THE "BLACK BOX"

BUILDING TRUST THROUGH EXPLAINABILITY



XAI FOSTERS CONFIDENCE, REDUCES RISKS, AND ENSURES ETHICAL & RESPONSIBLE AI DEPLOYMENT

SUMMARY OF GLOBAL REGULATIONS ON EXPLAINABLE AI (XAI)

EXISTING FRAMEWORKS AS OF JULY 2026

EUROPEAN UNION (EU)

EU AI ACT
(In Force, 2026)

- Risk-based approach
- **High-Risk Systems:** Strict XAI requirements (transparency, (transparency, right to explanation)
- **GPAI Models:** Specific transparency (Technical documentation)

CANADA

AIDA (Artificial Intelligence & Data Act)
(Implementation Stage)

- Focus on **High-Impact AI**
- Requirements for **explainability to individuals and responsible management**
- **Preventing Harmful Bias**

ETC. (Global Trends)

EMERGING FRAMEWORKS & GLOBAL STANDARDS

- Increasing international cooperation (G7, OECD)
- Work on global standards (ISO/IEC)
- Common themes of transparency and explainability
- Diverse national approaches (Brazil, India, etc.)

UNITED STATES (USA)

STATE-LEVEL & SECTORAL APPROACH

- **State Privacy Laws** (e.g., CA, CO, TX) with XAI requirements for automated decisions
- **Federal Sectoral Guidance** (FTC, EO 14110 directives)
- Fragmented but growing federal efforts

AUSTRALIA

EXISTING LAWS & GUIDELINES
(No omnibus AI law yet)

- Approach based on **Privacy Act** (updated AI rules) and other laws
- **AI Ethics Framework** (Voluntary Principles) and regulator guidance
- Focus on **safety & transparency**

CHINA

INTERRELATED SECTORAL & CONTENT REGULATIONS

- **Algorithmic Recommendation Regulations** (2021/2022)
- **Generative AI Interim Measures** (2023)
- **Focus on content control, safety assessments, and preventing discrimination**
- **PIPL** provisions for automated decisions

THE PROGRESS OF EXPLAINABLE AI (XAI) SINCE 2016

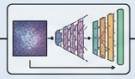
XAI DEVELOPMENT TRAJECTORY

EPOCH 3: 2022-PRESENT: MULTIMODAL, GENERATIVE, & DEPLOYED XAI

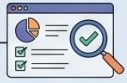
FOCUS: ADVANCED complexity, real-world deployment, and safety.

2022: Explainable LLMs
Chain-of-thought, rationalization

2023: Multimodal XAI
Combined Text+Image Explanation



2024: Generative AI transparency
Diffusion process visualization

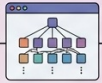


2024: Deployed XAI
Auditing, Governance, Regulatory Compliance (e.g., EU AI Act)

EPOCH 2: 2019-2021: INTERPRETABLE ARCHITECTURES & TRANSPARENCY

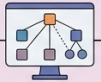
FOCUS: INTRINSIC interpretability and sercentricity.

2019: attention-based models
Self-Attention Visualization



2020: counterfactual explanations
Self-Attention Visualization

2020: counterfactual explanations
What-If Analysis

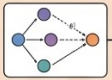


2020: conceptually interpretable models

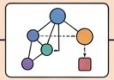
2021: XAI Evaluation Metrics
Benchmarks and Human Studies

FOCUS: INTRINSIC interpretability and user-centricity.

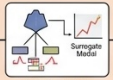
EPOCH 1: 2016-2018: POST-HOC FOUNDATIONS



2016: LIME
Local Surrogate Models



2017: CHAIP
Saliency Maps for CNNs



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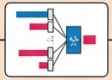
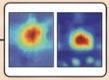


2017: SHAP
Shapley Values / Feature Attribution

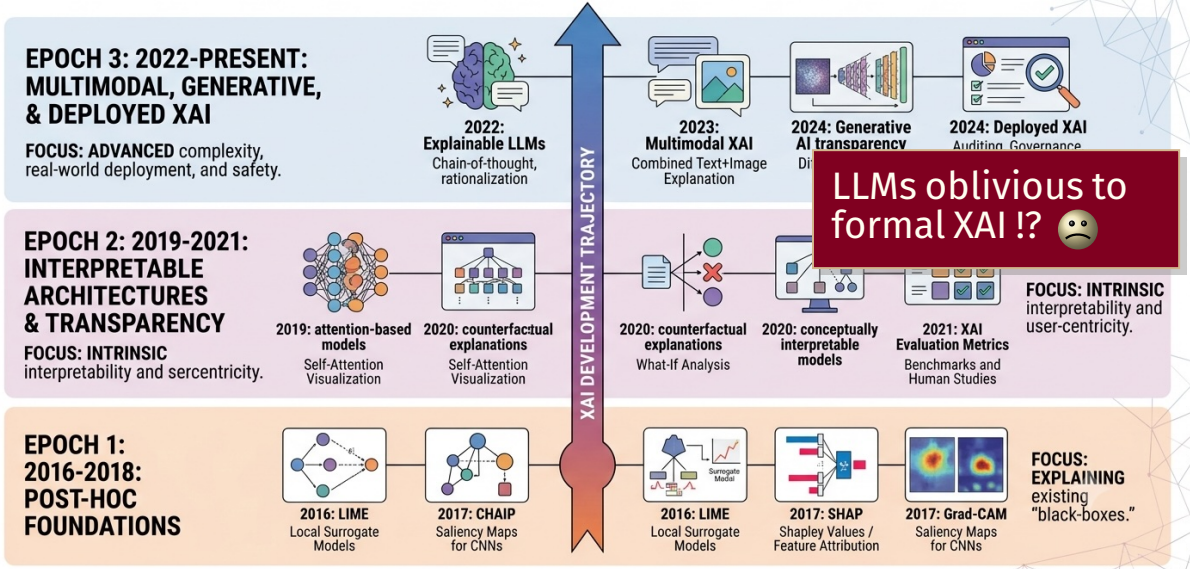


2017: Grad-CAM
Saliency Maps for CNNs

FOCUS: EXPLAINING existing "black-boxes."

Source: Synthesis of major XAI research trends.

THE PROGRESS OF EXPLAINABLE AI (XAI) SINCE 2016



Source: Synthesis of major XAI research trends.

EXPLAINABLE AI (XAI)

Making AI decisions understandable, trustworthy and actionable.

WHAT IS XAI?

Explainable AI aims to open the black box of AI systems by providing human-understandable reasons for their predictions.

WHY IT MATTERS

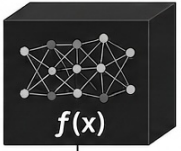
- Builds trust and transparency
- Supports fairness and accountability
- Helps detect errors and bias
- Enables better decisions and user acceptance

INPUT DATA

Age	45
Income	$60K
Education	Master
Credit Score	720
Debt	Low
...	...

BLACK BOX MODEL

(Complex ML/DL Model)



PREDICTION

Loan Approved

(Probability: 0.87)

XAI GOAL

Provide explanations for why the model made a specific prediction. Different explanations answer different needs.

TWO MAIN APPROACHES TO EXPLAIN A BLACK BOX

1 FEATURE SELECTION (Explanations as Rules)

Identify a small set of important features and generate human-readable rules that approximate the black box locally (for this prediction) or globally.

HOW IT WORKS

Select the most relevant features and learn simple rules that mimic the black box.

EXPLANATION (AS RULES)

IF Credit Score ≥ 700
AND Debt = Low
AND Income ≥ $50K

THEN Loan Approved
(Confidence: High)

Benefits: Easy to understand • Actionable • Supports compliance • Useful for communication and decision support

2 FEATURE ATTRIBUTION (Attribution Scores)

Quantify the contribution of each input feature to the specific prediction.

HOW IT WORKS

Estimate how much each feature pushed the prediction up or down from a baseline.

EXPLANATION (ATTRIBUTION SCORES)

Benefits: Shows influence of all features • Detects bias & spurious patterns • Supports debugging & model improvement

DIFFERENT EXPLANATIONS, DIFFERENT PURPOSES – SAME GOAL: UNDERSTAND AND TRUST AI

- **For Users & Decisions** — Use rules to justify and communicate decisions.
- **For Compliance** — Provide transparent, auditable justifications.
- **For Model Improvement** — Use attributions to find bias, errors, and issues.
- **For Domain Experts** — Combine insights with domain knowledge.

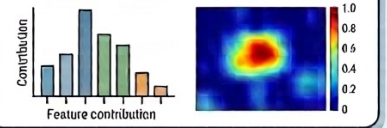
XAI builds trust and enables responsible, human-centered AI.

CATEGORIZING NON-SYMBOLIC XAI

FEATURE ATTRIBUTION

Method: Explains the **contribution** of each feature to a prediction.

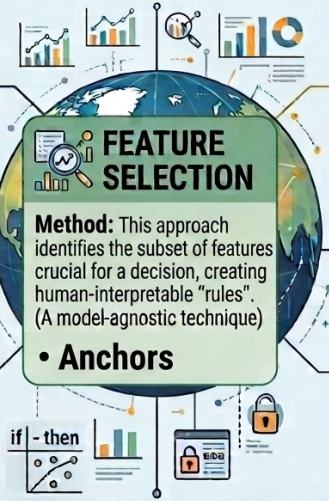
- **LIME** (Local Interpretable Model-agnostic Explanations)
- **SHAP** (SHapley Additive exPlanations)
- **Saliency Maps** (for visual models)



FEATURE SELECTION

Method: This approach identifies the subset of features crucial for a decision, creating human-interpretable "rules". (A model-agnostic technique)

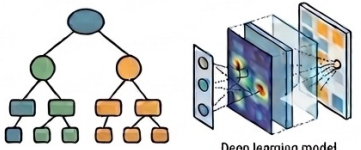
- **Anchors**



INTRINSIC INTERPRETABILITY

Method: Designing models with understandable **internal** logic.

- **DTs** (Decision Trees)
- **DLs** (Decision Lists), ...



⚠ Core premises of non-symbolic XAI disproved ⚠

[MH24, HM24, IIM22, Ign20]

[RSG16, LL17, RSG18, Rud19]

LIME

"Why Should I Trust You?"
Explaining the Predictions of Any Classifier

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X

SHAP

A Unified Approach to Interpreting Model Predictions

X

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PERSPECTIVE
https://doi.org/10.1038/s42256-019-0048-x
nature machine intelligence

Stop explaining black box machine learning models for high stakes decisions and use interpretable models instead

X

Cynthia Rudin

Intrinsic Interpretability

Anchors: High-Precision Model-Agnostic Explanations

X

Anchor

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Anchor

Rigorous explainability with LIME/SHAP/Anchors is a myth !!!

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[MH24, HM24, IIM22, Ign20]

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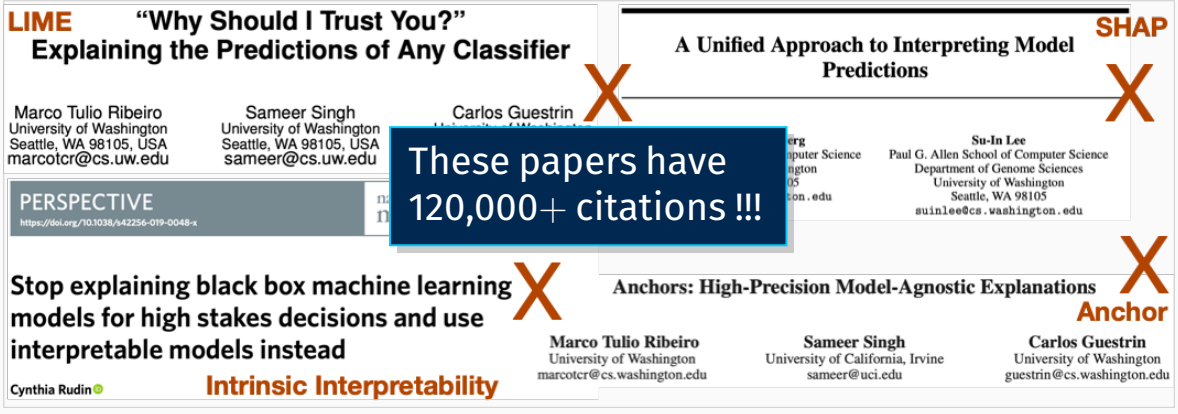

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My invited talks at FLoC 2026...

- Invited lecture at SAT/SMT/AR Summer School:
  "Problem Solving with SAT Oracles" Jul 13
- Invited talk at the SAT 2026 conference:
  "Trustable Explainable AI: SAT to the Rescue" Jul 21
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Feature selection

Talks on formal XAI cover the myths of non-symbolic XAI

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Feature selection

Talks on formal XAI cover the myths of non-symbolic XAI

Quoting M. Vardi, from a talk some years ago...

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Outline

Definitions – classification problems

- Set of features $\mathcal{F} = \{1, 2, \ldots, m\}$, each feature i taking values from domain $\mathbb{D}_i$
  - Features can be categorical, discrete or real-valued
  - Feature space: $\mathbb{F} = \Pi_{i=1}^{m} \mathbb{D}_i$
- Set of classes $\mathcal{K} = \{c_1, \ldots, c_K\}$
- ML model $\mathbb{M}$ computes a (non-constant) classification function $\kappa : \mathbb{F} \to \mathcal{K}$
- Instance $(\mathbf{v}, c)$, for point $\mathbf{v} = (v_1, \ldots, v_m) \in \mathbb{F}$, with prediction $c = \kappa(\mathbf{v})$, $c \in \mathcal{K}$
  - **Goal:** compute explanations for $(\mathbf{v}, c)$

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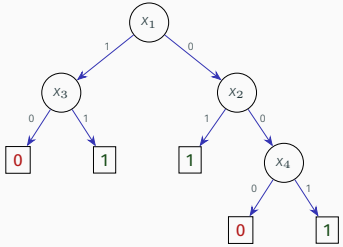
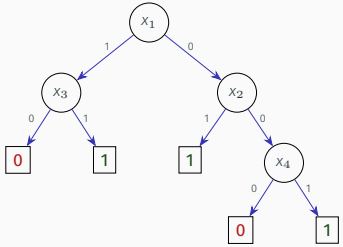
  - **Goal:** compute explanations for $(\mathbf{v}, c)$
- **Framework can be extended to regression problems** [MS24]
  - In this talk: κ replaced by ρ for regression problems...

Symbolic explanations – intuition



- Domains: $\mathbb{D}_i = \{0, 1\}, i = 1, 2, 3, 4$
- Feature space: $\mathbb{F} = \{0, 1\}^4$
- Classes: $\mathcal{K} = \{0, 1\}$
- κ defined by DT
- Instance: $((0, 0, 1, 1), 1)$

Symbolic explanations – intuition



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- **Abductive explanation** answers question "**Why** (the prediction)?" as a **rule**: IF <COND> THEN $\kappa(\mathbf{x}) = c$
  - E.g. IF $\neg x_1 \wedge \neg x_2 \wedge x_4$ THEN $\kappa(\mathbf{x}) = 1$

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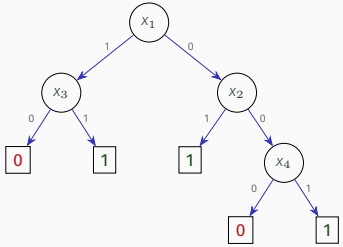

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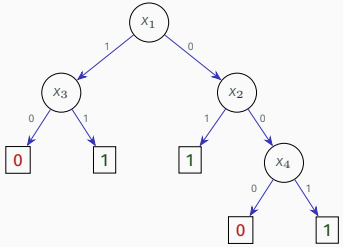
  - E.g. IF $\neg x_1 \wedge \neg x_2 \wedge x_4$ THEN $\kappa(\mathbf{x}) = 1$
- **Contrastive explanation** answers question "**Why Not** (some other prediction)?"
  - E.g. $\neg x_1 \wedge \neg x_2 \wedge x_3 \wedge \neg x_4$ is consistent with $\kappa(\mathbf{x}) = 0$

Symbolic explanations – intuition



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- **Abductive explanation** answers question "**Why** (the prediction)?" as a **rule**: IF <COND> THEN $\kappa(\mathbf{x}) = c$
  - E.g. IF $\neg x_1 \wedge \neg x_2 \wedge x_4$ THEN $\kappa(\mathbf{x}) = 1$
- **Contrastive explanation** answers question "**Why Not** (some other prediction)?"
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- Explanations represented by sets of features (other details left implicit)
  - E.g. $\{1, 2, 4\}$ or $\{4\}$

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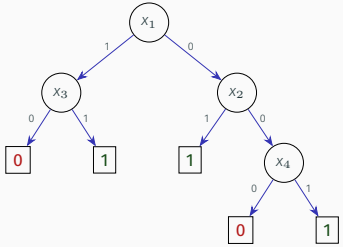
Consistency decided with automated reasoners (SAT, SMT, ...)

An example



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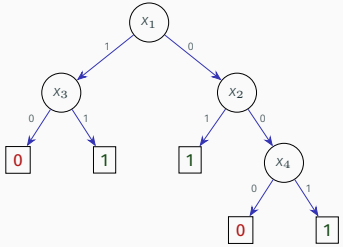

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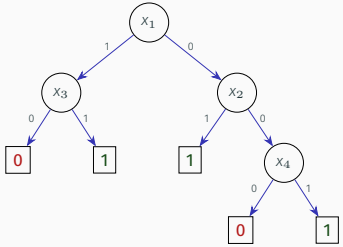

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Hitting-set duality used ubiquitously!

Logic-based abduction vs. abductive explanations

Logic-based abduction

[EG95]

- Tuple: $\mathcal{P} = \langle V, H, M, R \rangle$
  - V: Finite set of propositional variables
  - H: Set of hypotheses
  - $M \subseteq V$: Set of manifestations
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- A solution for $\mathcal{P}$ is subset-minimal $S \subseteq H$ such that,
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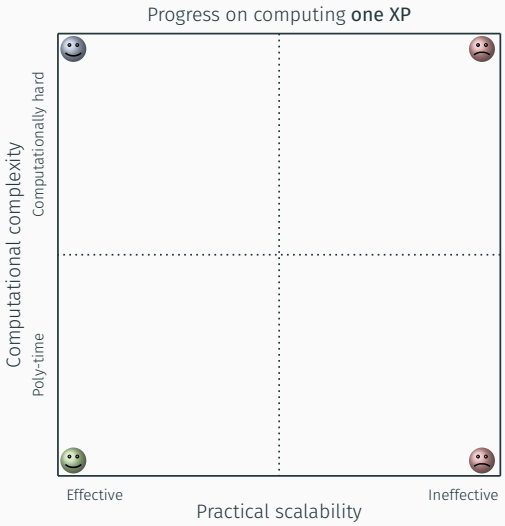

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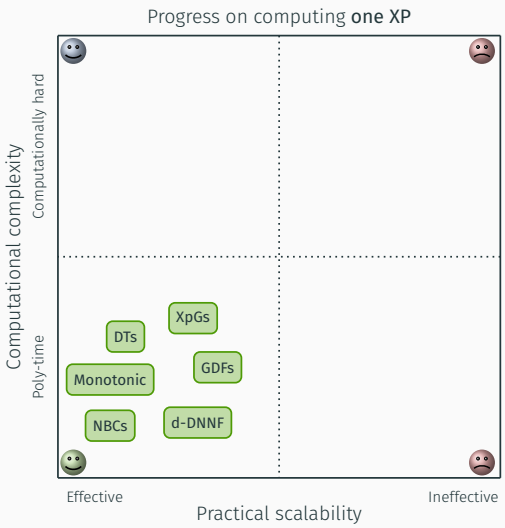

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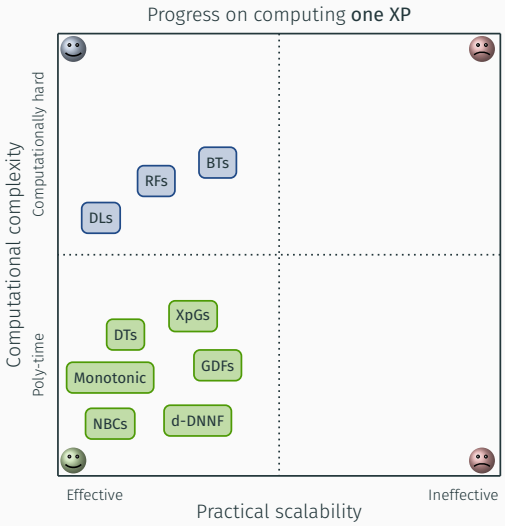

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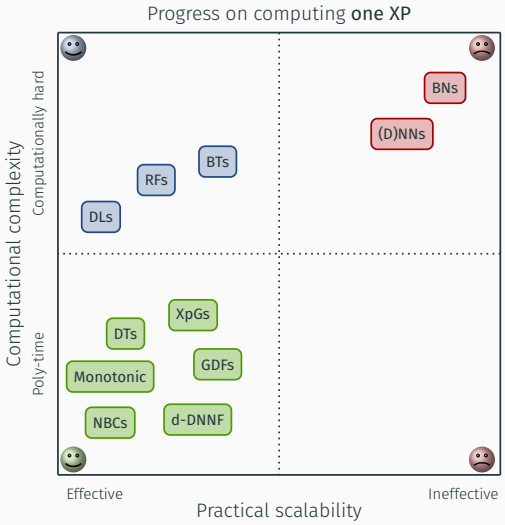

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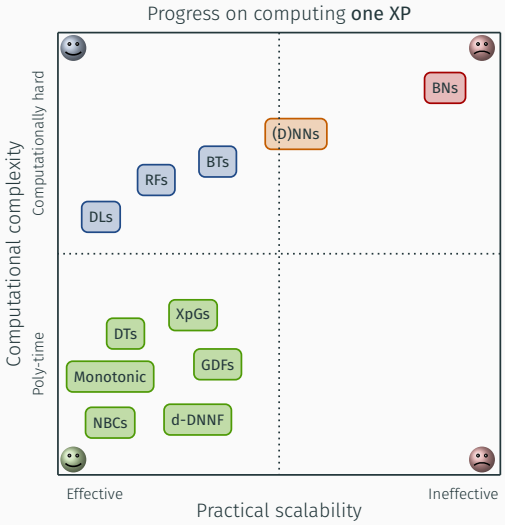
    - Bayesian networks (BNs) [SCD18]

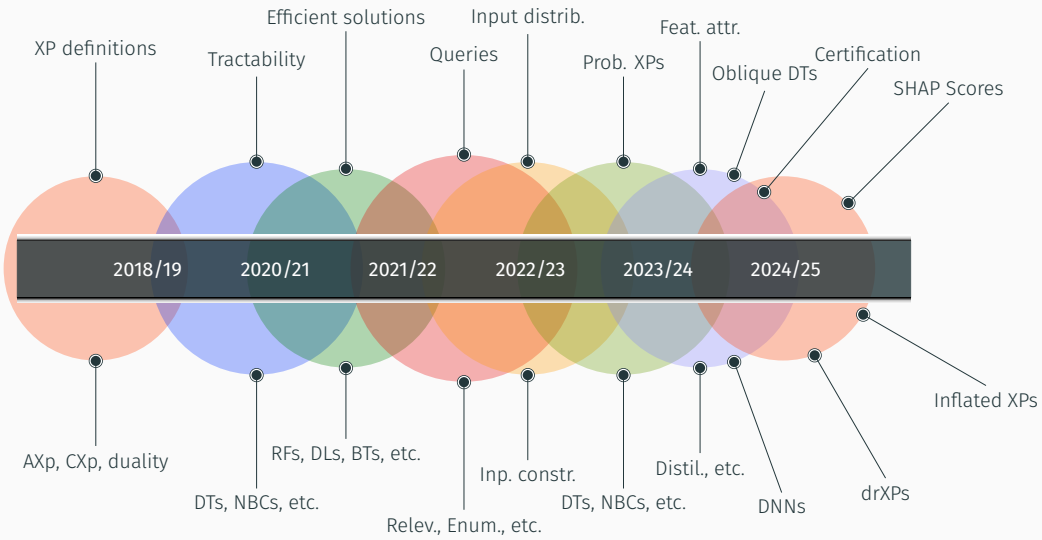
Progress in symbolic XAI – recent progress

[INM19c, IIM20, MGC+20, MGC+21, HIIM21, IMS21, IM21, CM21, IIM22, HII+22, IISMS22, HM23a, IHM+24a]



- Advances up until 2024:
  - Polynomial-time:
    - Naive-Bayes classifiers (NBCs) [MGC+20]
    - Decision trees (DTs) [IIM20, HIIM21, IIM22]
    - XpG's: DTs, OBDDs, OMDDs, etc. [HIIM21]
    - Monotonic classifiers [MGC+21]
    - Propositional languages (e.g. d-DNNF, ...) [HII+22]
    - Additional results [CM21, HII+22, CM23, BKR+25]
  - Comp. hard, but effective (efficient in practice):
    - Random forests (RFs) [IMS21, IISMS22]
    - Decision lists (DLs) [IM21]
    - Boosted trees (BTs) [INM19c, IISMS22]
  - Comp. hard, but some practical scalability:
    - Neural networks (NNs) [HM23a, IHM+24a, IHM+24b]
  - Comp. hard, and ineffective (hard in practice):
    - Bayesian networks (BNs) [SCD18]

The emergence of symbolic explainability – timeline



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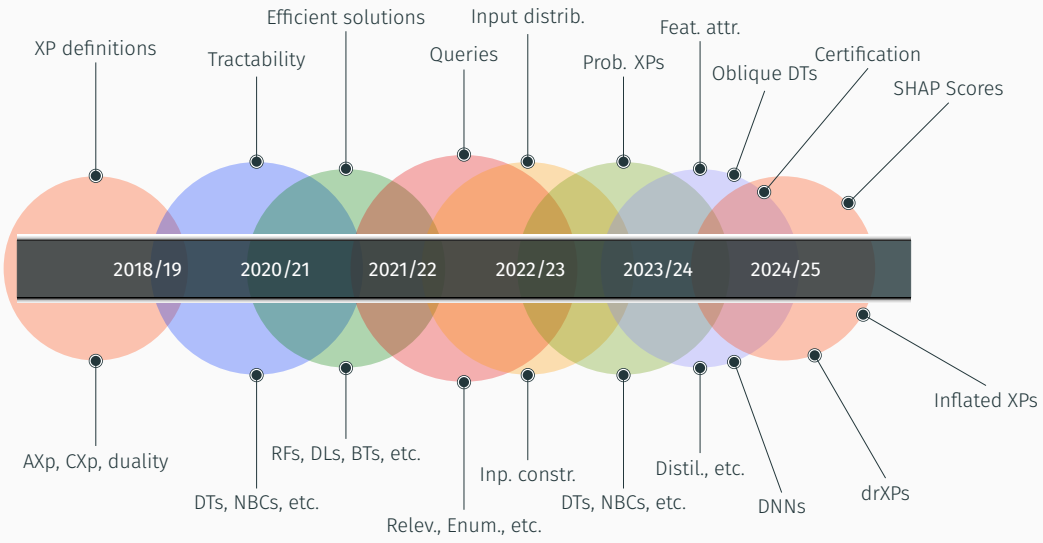

Since 2025: Corrected SHAP Scores, Sample-based Xps, Certification, Rule-ensembles, other ML models, etc.

A key ingredient – feature relevancy

[HIIM21]

- Consider instance $(\mathbf{v}, c)$
- Sets of all AXp's & CXp's:

$$\mathbb{A} \coloneqq \{\mathcal{X} \subseteq \mathcal{F} \,|\, \mathsf{AXp}(\mathcal{X})\}$$

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An example

- Consider the classifier:

$$\kappa(x_1, x_2, x_3, x_4) = \bigvee_{i=1}^{4} x_i$$

  - $(\mathbf{v}, c) = ((0, 0, 0, 1), 1)$

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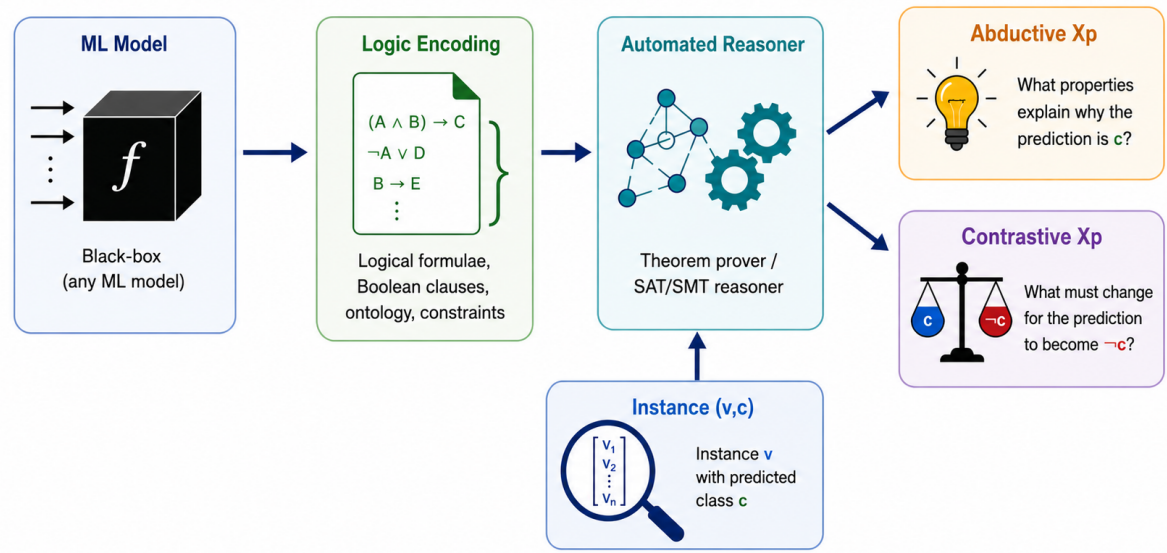
- Features 1, 2, 3 are irrelevant, since there are not included in any AXp/CXp

  - Obs: irrelevant features are absolutely unimportant!

    We could propose some other explanation by adding features 1, 2 or 3 to AXp $\{4\}$, but prediction would remain unchanged for any value assigned to those features

    - And we aim for irreducibility (Occam's razor is a mainstay of AI/ML)

Model-Aware Explanations

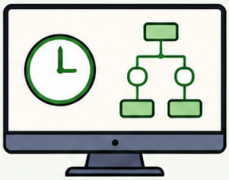


Model-Aware Explanations – Poly-Time

ML Model

Black-box
(any ML model)

Poly-time Algorithm



Polynomial-time
algorithm that computes
explanations

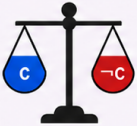
Instance (v, c)

Instance **v**
with predicted
class **c**

Abductive Xp

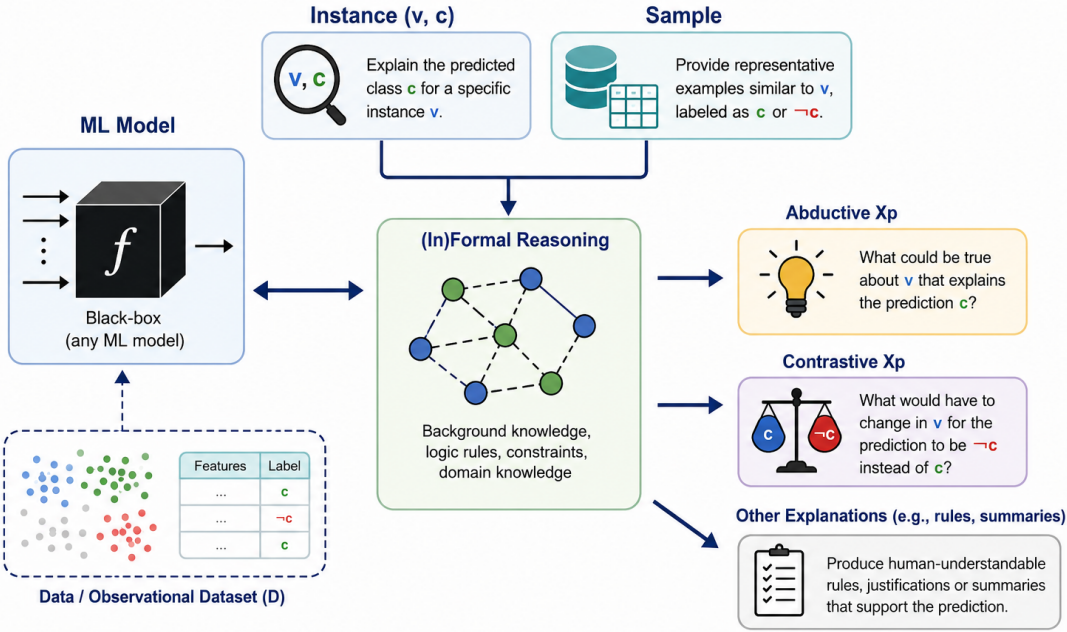
What properties
explain why the
prediction is **c**?

Contrastive Xp



What must change
for the prediction
to become ¬c?

Model-Agnostic Explanations



Another key ingredient – model-agnostic sample-based XAI

[MLM25]

- A sample $\mathbb{S}$ is a subset of feature space $\mathbb{F}$; a prediction p_i is associated with each $\mathbf{v}_i \in \mathbb{S}$
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$$\text{WeakAXp}(\mathcal{X}) \quad := \quad \forall(\mathbf{x} \in \mathbb{S}). \bigwedge\nolimits_{j \in \mathcal{X}} (x_j = v_j) \rightarrow (\kappa(\mathbf{x}) = c)$$

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[IISM24]

NOTE: the sample can be your training data and/or you can sample the ML model as much as you wish!

Example of a sample, with instance $((1, 1, 1, 1), 1)$

#	x_1	x_2	x_3	x_4	$\kappa(\cdot)$
00	0	0	0	0	0
02	0	0	1	0	0
03	0	0	1	1	0
06	0	1	1	0	0
08	1	0	0	0	1
12	1	1	0	0	1
15	1	1	1	1	1

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- Move from row id 15 to one row taken out of $\{00, 02, 03, 06\}$

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$\therefore$ sample-based explainability can use the algorithms developed for DTs!

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- And formal explanations are rigorous!
  - **Obs:** Anchors can produce **incorrect** explanations (talk on Tue...)

Outline

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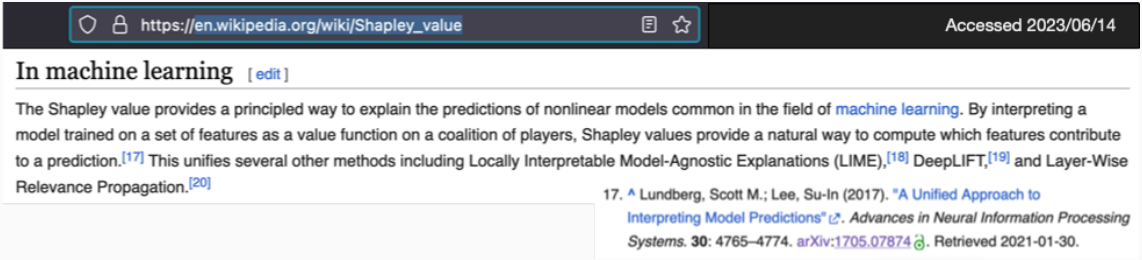

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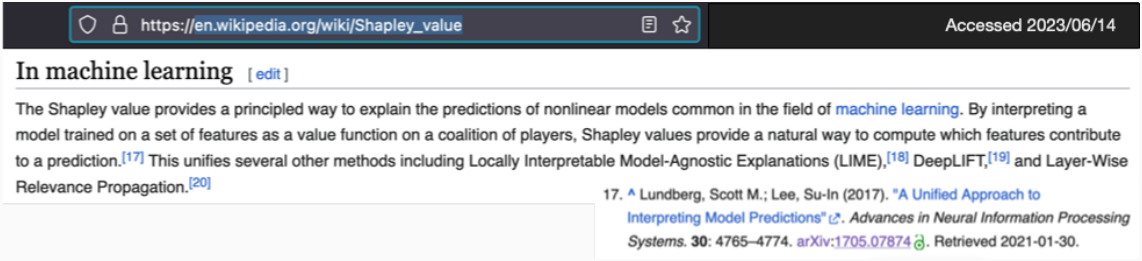

- Q: Do Shapley values for XAI really provide a rigorous measure of feature importance?

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- $\phi\colon 2^{\mathcal{F}} \to \mathbb{R}$ defined by,

$$\phi(\mathcal{S}) = 1/2^{|\mathcal{F} \setminus \mathcal{S}|} \sum_{\mathbf{x} \in \Upsilon(\mathcal{S})} \kappa(\mathbf{x}) \; = v_e(\mathcal{S})$$

  $\phi(\mathcal{S})$ represents the expected value of the classifier on the points given by $\Upsilon(\mathcal{S})$

How are Shapley values used in explainability?

- Instance: $(\mathbf{v}, c)$
- $\Upsilon\colon 2^{\mathcal{F}} \to 2^{\mathbb{F}}$ defined by, [ABBM21, ABBM23]

$$\Upsilon(\mathcal{S}) = \{\mathbf{x} \in \mathbb{F} \mid \wedge_{i \in \mathcal{S}} x_i = v_i\}$$

  $\Upsilon(\mathcal{S})$ gives points in feature space having the features in $\mathcal{S}$ fixed to their values in $\mathbf{v}$

- $\phi\colon 2^{\mathcal{F}} \to \mathbb{R}$ defined by,

$$\phi(\mathcal{S}) = 1/2^{|\mathcal{F}\setminus\mathcal{S}|} \sum_{\mathbf{x} \in \Upsilon(\mathcal{S})} \kappa(\mathbf{x}) \ = v_e(\mathcal{S})$$

  $\phi(\mathcal{S})$ represents the expected value of the classifier on the points given by $\Upsilon(\mathcal{S})$

- Sc: $\mathcal{F} \to \mathbb{R}$ defined by,

$$\mathrm{Sc}(i) = \sum_{\mathcal{S} \subseteq (\mathcal{F}\setminus\{i\})} |\mathcal{S}|!(|\mathcal{F}| - |\mathcal{S}| - 1)!/|\mathcal{F}|! \times (\phi(\mathcal{S} \cup \{i\}) - \phi(\mathcal{S}))$$

  For all subsets of features, excluding i, compute the expected value of the classifier, with and without i fixed, weighted by $\frac{1}{n}\binom{n}{|\mathcal{S}|}^{-1}$

  - **Obs:** Uniform distribution assumed; it suffices for our purposes

How are Shapley values used in explainability?

Marginal contribution (in SHAP lingo)!

- Instance: $(\mathbf{v}, c)$
- Υ: $2^{\mathcal{F}} \to 2^{\mathbb{F}}$ defined by, [ABBM21, ABBM23]

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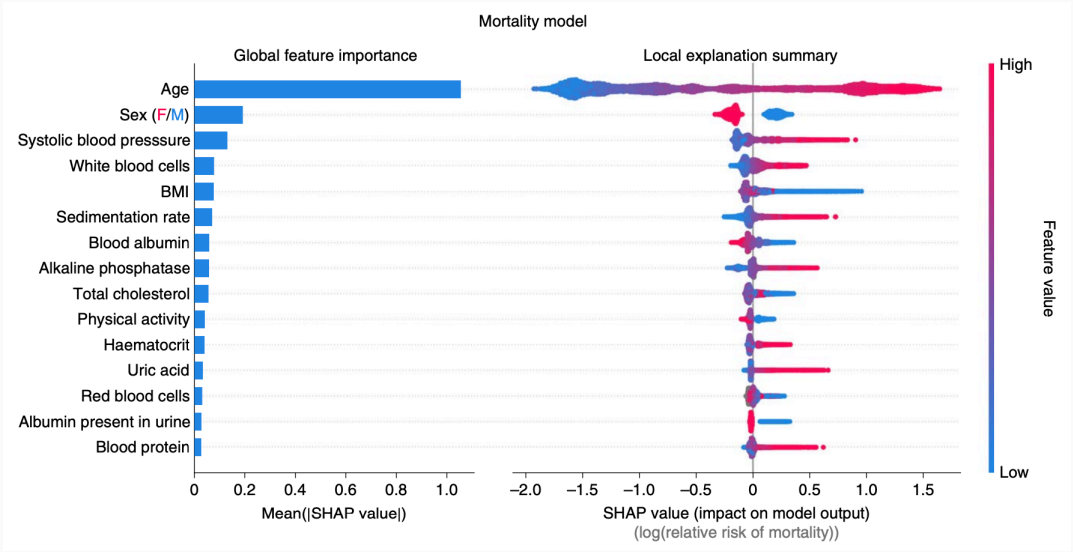
How are Shapley values computed in practice?

- Exact evaluation is computationally (very) hard [VLSS21, ABBM21, VLSS22, ABBM23, HM24]
- SHAP proposes a sample-based approach; with no guarantees of rigor [LL17]
  - Recent experiments revealed little to no correlation between Shapley values and SHAP's results [HM23b]
- Polynomial-time algorithm for deterministic decomposable boolean circuits [ABBM21]
- Polynomial-time algorithm for boolean functions represented with a truth-table [HM23b]

How are Shapley values computed in practice? example SHAP output...



What do Shapley values tell in terms of feature importance?

- [SK10] reads:
  "*According to the 2nd axiom, if two features values have an identical influence on the prediction they are assigned contributions of equal size. The 3rd axiom says that if a* ***feature has no influence*** *on the prediction* ***it is assigned a contribution of 0****.*"
  (Obs: the axioms refer to the axiomatic characterization of Shapley values.)

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- **Obs:** Shapley values are defined axiomatically, i.e. **no** immediate relationship with AXp's/CXp's or with feature (ir)relevancy
  - **Qs**: can we have **irrelevant** features with a non-zero Shapley value, and/or **relevant** features with a Shapley of zero?
    - Recall: **relevant** features occur in **some** AXp/CXp; **irrelevant** features do **not** occur in **any** AXp/CXp

Shapley values vs. feature (ir)relevancy – identified issues

[HM23b, HM23c, HM23d, MH23, HM24, MH24]

- Boolean classifier, instance $(\mathbf{v}, c)$, and some $i, i_1, i_2 \in \mathcal{F}$:

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  - Issue I3 occurs if,
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  - Issue I4 occurs if,
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Shapley values vs. feature (ir)relevancy – identified issues [HM23b, HM23c, HM23d, MH23, HM24, MH24]

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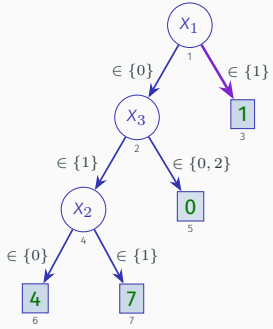
Any of these issues is a cause of (**serious**) concern per se**!**

Some stats – all boolean functions with 4 variables

[HM23b, HM23c, HM23d, MH23, HM24, MH24]

Issue-related metric	Value	Recap issue
# of functions	65536	
# number of instances	1048576	
# of I1 issues	781696	
# of functions with I1 issues	65320	
% I1 issues / function	99.67	[Irrelevant(i) $\wedge$ (Sv(i) $\neq 0$)]
# of I2 issues	105184	
# of functions with I2 issues	40448	
% I2 issues / function	61.72	[Irrelevant(i_1) $\wedge$ Relevant(i_2) $\wedge$ ($\lvert$Sv(i_1)$\rvert$ > $\lvert$Sv(i_2)$\rvert$)]
# of I3 issues	43008	
# of functions with I3 issues	7800	
% I3 issues / function	11.90	[Relevant(i) $\wedge$ (Sv(i) $= 0$)]
# of I4 issues	5728	
# of functions with I4 issues	2592	
% I4 issues / function	3.96	[Irrelevant(i_1) $\wedge$ (Sv(i_1) $\neq 0$)] $\wedge$ [Relevant(i_2) $\wedge$ (Sv(i_2) $= 0$)]
# of I5 issues	1664	
# of functions with I5 issues	1248	
% I5 issues / function	1.90	[Irrelevant(i) $\wedge \forall_{1 \leq j \leq m, j \neq i}$ ($\lvert$Sv(j)$\rvert$ < $\lvert$Sv(i)$\rvert$)]

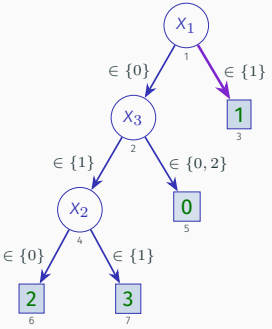
Previous results do matter! Let's go non-boolean...



DT1

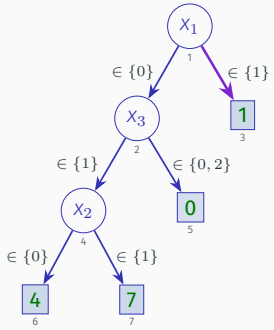
row #	x_1	x_2	x_3	$\kappa_1(\mathbf{x})$	$\kappa_2(\mathbf{x})$
1	0	0	0	0	0
2	0	0	1	4	2
3	0	0	2	0	0
4	0	1	0	0	0
5	0	1	1	7	3
6	0	1	2	0	0
7	1	0	0	1	1
8	1	0	1	1	1
9	1	0	2	1	1
10	1	1	0	1	1
11	1	1	1	1	1
12	1	1	2	1	1

Tabular representations



DT2

Instance $((1, 1, 2), 1)$ – which feature matters the most for prediction 1?



DT1

row #	x_1	x_2	x_3	$\kappa_1(\mathbf{x})$	$\kappa_2(\mathbf{x})$
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2	0	0	1	4	2
3	0	0	2	0	0
4	0	1	0	0	0
5	0	1	1	7	3
6	0	1	2	0	0
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9	1	0	2	1	1
10	1	1	0	1	1
11	1	1	1	1	1
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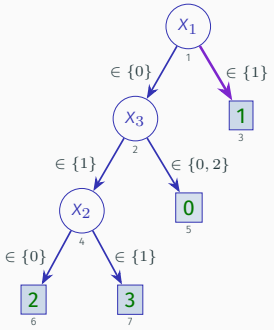
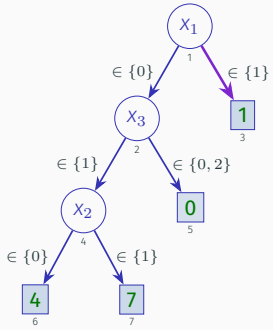
Tabular representations



DT2

Computing XPs – make sense...



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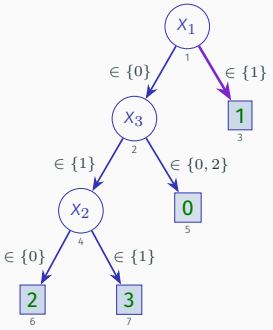
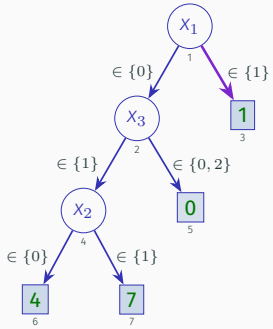
Tabular representations



DT2

XPs: AXps/CXps		
DT	AXps	CXps
DT1	$\{1\}$	$\{1\}$
DT2	$\{1\}$	$\{1\}$

Computing XPs, AEs – also make sense...



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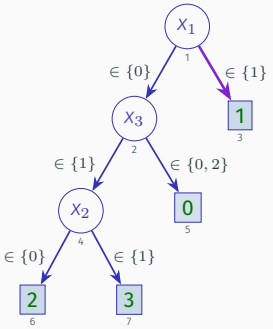

DT2

XPs: AXps/CXps		
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DT1	$\{1\}$	$\{1\}$
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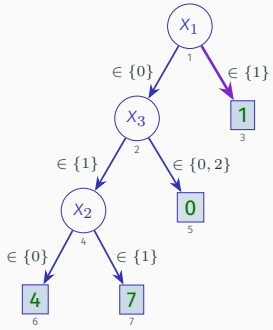
Adversarial examples	
DT	minimal AEs
DT1	$\{1\}$
DT2	$\{1\}$

Computing XPs, AEs & Svs



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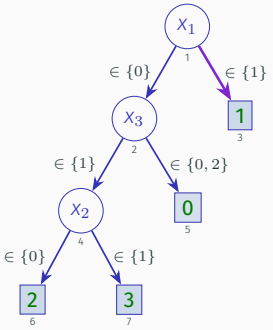

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Adversarial examples	
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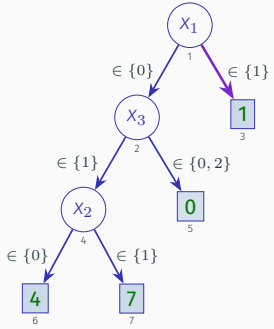
Shapley values			
DT	Sc(1)	Sc(2)	Sc(3)
DT1	0.000	0.083	-0.500
DT2	0.278	0.028	-0.222

Computing XPs, AEs & Svs – what!?!?



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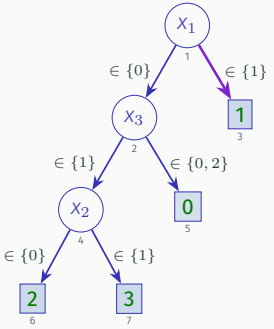

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Shapley values in theory				
DT	Sc(1)	Sc(2)	Sc(3)	
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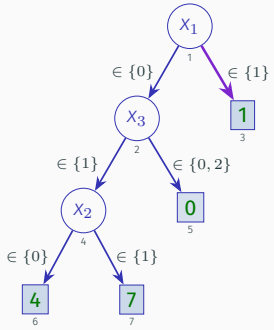
Computing XPs, AEs & Svs – what!?!?



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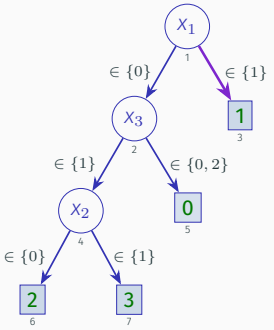

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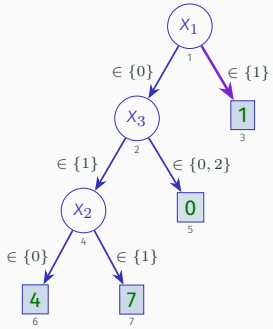
Computing XPs, AEs & Svs – what!?!?

∴ Shapley values can mislead human decision-makers !



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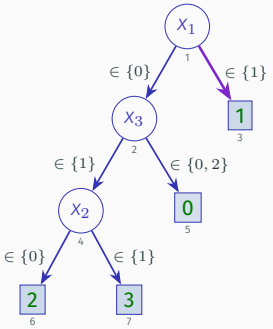
12	1	1	2	1	1

Tabular representations



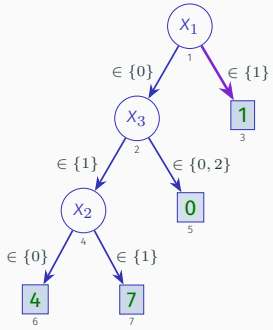
DT2

XPs: AXps/CXps		
DT	AXps	CXps
DT1	$\{1\}$	$\{1\}$
DT2	$\{1\}$	$\{1\}$

Adversarial examples	
DT	minimal AEs
DT1	$\{1\}$
DT2	$\{1\}$

Shapley values in theory				
DT	Sc(1)	Sc(2)	Sc(3)	
DT1	0.000	0.083	-0.500	!!!
DT2	0.278	0.028	-0.222	!!

Computing XPs, AEs & Svs – what!?!?

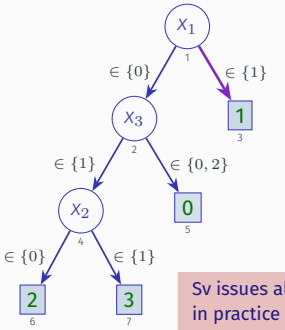


DT1

row #	x_1	x_2	x_3	$\kappa_1(\mathbf{x})$	$\kappa_2(\mathbf{x})$
1	0	0	0	0	0
2	0	0	1	4	2
3	0	0	2	0	0
4	0	1	0	0	0
5	0	1	1	7	3
6	0	1	2	0	0
7	1	0	0	1	1
8	1	0	1	1	1
9	1	0	2	1	1
10	1	1	0	1	1
11	1	1	1	1	1
12	1	1	2	1	1

Tabular representations

$\therefore$ Shapley values can mislead human decision-makers !



Sv issues also occur in practice [HM23d]

DT2

XPs: AXps/CXps

DT	AXps	CXps
DT1	$\{1\}$	$\{1\}$
DT2	$\{1\}$	$\{1\}$

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Shapley values in theory

DT	Sc(1)	Sc(2)	Sc(3)	
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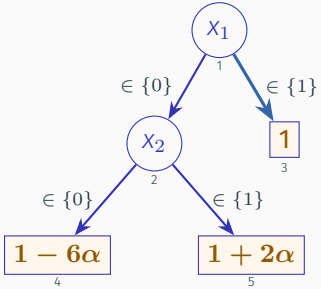
Another example – arbitrary mistakes!

[LHAM24]



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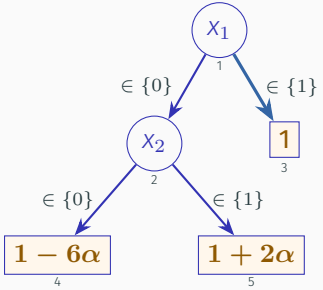

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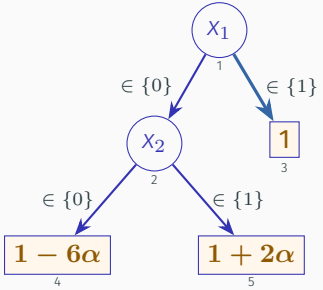

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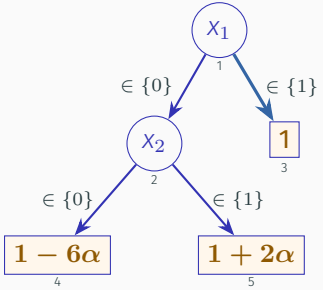

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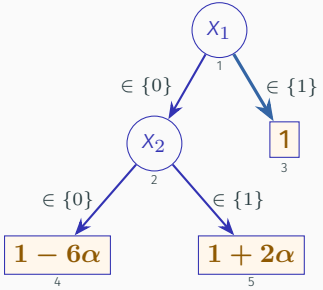

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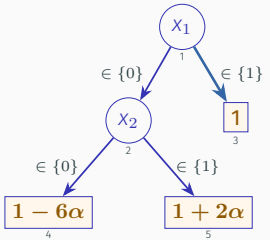
Note: example devised in 2024 by O. Letoffe, PhD student at IRIT

More detail

row	x_1	x_2	$\rho(\mathbf{x})$	$\rho_a(\mathbf{x})$ $\alpha = 1/2$	$\rho_b(\mathbf{x})$ $\alpha = 1/4$
1	0	0	$1-6\alpha$	-2	$-1/2$
2	0	1	$1+2\alpha$	2	$3/2$
3	1	0	1	1	1
4	1	1	1	1	1



$\mathcal{S}$	rows($\mathcal{S}$)	$v_e(\mathcal{S})$
$\emptyset$	$1,2,3,4$	$1-\alpha$
$\{x_1\}$	$3,4$	1
$\{x_2\}$	$2,4$	$1+\alpha$
$\{x_1,x_2\}$	4	1

$i = 1$

$\mathcal{S}$	$v_e(\mathcal{S})$	$v_e(\mathcal{S}\cup\{1\})$	$\Delta_1(\mathcal{S})$	$\varsigma(\mathcal{S})$	$\varsigma(\mathcal{S})\times\Delta_1(\mathcal{S})$
$\emptyset$	$1-\alpha$	1	α	$1/2$	$\alpha/2$
$\{2\}$	$1+\alpha$	1	$-\alpha$	$1/2$	$-\alpha/2$
			$\mathsf{Sc}_E(1)$	$=$	0

$i = 2$

$\mathcal{S}$	$v_e(\mathcal{S})$	$v_e(\mathcal{S}\cup\{2\})$	$\Delta_2(\mathcal{S})$	$\varsigma(\mathcal{S})$	$\varsigma(\mathcal{S})\times\Delta_2(\mathcal{S})$
$\emptyset$	$1-\alpha$	$1+\alpha$	2α	$1/2$	α
$\{1\}$	1	1	0	$1/2$	0
			$\mathsf{Sc}_E(2)$	$=$	α

SHAP scores also fail with regression models

[LHM24b]

- Let $\mathcal{F} = \{1, 2\}$, $\mathbb{D}_1 = \mathbb{D}_2 = \mathbb{D} = [-1/2, 3/2]$, $\mathbb{F} = \mathbb{D} \times \mathbb{D}$
  - Also, let $\mathbb{D}^+ = [1/2, 3/2]$ and $\mathbb{D}^- = \mathbb{D} \setminus \mathbb{D}^+$

- Regression model maps to real values, i.e. $\mathbb{K} = \mathbb{R}$:

$$\rho_2(x_1, x_2) = \begin{cases} x_1 & \text{if } x_1 \in \mathbb{D}^+ \\ x_2 - 2 & \text{if } x_1 \notin \mathbb{D}^+ \wedge x_2 \notin \mathbb{D}^+ \\ x_2 + 1 & \text{if } x_1 \notin \mathbb{D}^+ \wedge x_2 \in \mathbb{D}^+ \end{cases}$$

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- Average values unchanged (wrt previous example), and so $\mathrm{Sc}(1) = 0$ and $\mathrm{Sc}(2) = \alpha$

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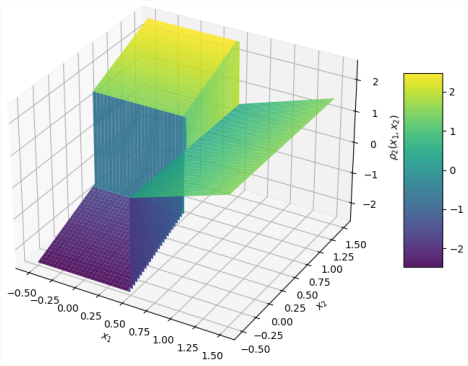

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SHAP scores fail even when Lipschitz continuity holds!

[LHM24b]

$$\rho_3(x_1, x_2) = \begin{cases} x_1 & \text{if } x_2 \leq 1 \wedge \alpha x_1 \leq \alpha \\ (1 + 4|\alpha|)x_1 - 4|\alpha| & \text{if } x_2 \leq 1 \wedge \alpha x_1 \geq \alpha \\ 28|\alpha|x_1x_2 + (1 - 28|\alpha|)x_1 - 28|\alpha|x_2 + 28|\alpha| & \text{if } x_2 \geq 1 \wedge \alpha x_1 \leq \alpha \\ -4|\alpha|x_1x_2 + (1 + 8|\alpha|)x_1 + 4|\alpha|x_2 - 8|\alpha| & \text{if } x_2 \geq 1 \wedge \alpha x_1 \geq \alpha \end{cases}$$

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NOTE: **SHAP scores also fail with arbitrary differentiability!**

Outline

Corrected SHAP scores & feature importance scores

[LHM24a, LHAM24]

- Is the theory of Shapley values incorrect?

Corrected SHAP scores & feature importance scores

[LHM24a, LHAM24]

- Is the theory of Shapley values incorrect? No!

Corrected SHAP scores & feature importance scores

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- What is flawed is the characteristic function used in XAI [SK10, SK14, LL17]
  - In XAI: characteristic function uses the expected value
  - This defines the *marginal contribution* in SHAP lingo...

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  - Resulting scores are (still) Shapley values & identified issues no longer observed
- Observed tight connection between feature attribution and power indices from a priori voting power

An initial compromise

[LHAM24]

- Replace the characteristic function used for SHAP scores:

$$v_e(\mathcal{S}) \quad := \quad \mathbf{E}[\tau(\mathbf{x}) \,|\, \mathbf{x}_{\mathcal{S}} = \mathbf{v}_{\mathcal{S}}]$$

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- Issues with non-boolean classifiers disappear; issues with boolean classifiers remain
- Developed SSHAP prototype using SHAP's code base [LHM24a]

Fixing the known issues of SHAP scores

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- Known issues of SHAP scores guaranteed **not** to occur (more later)

- **Corrected** SHAP scores reveal tight connection between XAI by feature selection (i.e. WAXps) and feature attribution

Outline

- Shapley values estimated with well-known algorithm – CGT [CGT09]
  - Strong theoretical guarantees on approximation of Shapley values
  - With enough sampling, given by ϵ and α:

$$\text{Prob}[|\hat{\text{Sc}}(i) - \text{Sc}(i)| \leq \epsilon] \geq 1 - \alpha$$

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  - Model-aware WAXp checking unrealistic for highly complex models
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A glimpse of NuSHAP

[MHL25]

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- Replace expected value with novel characteristic function: test for WAXp
  - Model-aware WAXp checking unrealistic for highly complex models
    - And CGT may require a very large number of WAXp checks
  - NuSHAP uses model-agnostic (and sample-based) WAXp checking
    - Polynomial-time algorithm
    - Flexibility in the sample chosen, e.g. training data

NuSHAP vs. *SHAP

- Compute SHAP scores with NuSHAP and SHAP
- Compare features' rankings using Rank-Biased Overlap (RBO) [WMZ10]
  - Give weight to differences in first k elements of ranking
  - E.g. $\text{RBO}([0,1,2,3,4,5,6,7,8,9],[0,1,2,3,4,5,6,7,8,9],k=5)=1.0$
- Results for different datasets and ML models (using scikit, XGBoost):
  - LR: (adult, corral); DT: (iris, mux6); kNN: (connect_4, spambase, spectf); BT: (clean1, coil2000, dna); CNN: (MNIST)

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		adult	corral	iris	mux6	connect_4	spambase	spectf	clean1	coil2000	dna	MNIST
Min	nuSHAP vs. SHAP	0.08	0.17	0.31	0.32	0.0	0.01	0.0	0.0	0.0	0.0	0.0
	nuSHAP vs. |SHAP|	0.05	0.12	0.27	0.32	0.0	0.05	0.0	0.0	0.0	0.03	0.0
Max	nuSHAP vs. SHAP	0.96	0.96	0.94	0.97	0.9	0.94	0.91	0.69	0.69	0.88	0.06
	nuSHAP vs. |SHAP|	0.88	0.97	0.94	0.95	0.77	0.94	0.91	0.88	0.69	0.88	0.06
Mean	nuSHAP vs. SHAP	0.37	0.53	0.84	0.7	0.21	0.41	0.2	0.12	0.05	0.17	0.0
	nuSHAP vs. |SHAP|	0.31	0.5	0.84	0.69	0.19	0.42	0.19	0.17	0.08	0.43	0.0

NuSHAP vs. *SHAP – run times

- Compute SHAP scores with NuSHAP and SHAP
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	nuSHAP vs. |SHAP|	0.05	0.12	0.27	0.32	0.0	0.05	0.0	0.0	0.0	0.03	0.0
Max	nuSHAP vs. SHAP	0.96	0.96	0.94	0.97	0.9	0.94	0.91	0.69	0.69	0.88	0.06
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Mean	nuSHAP vs. SHAP	0.37	0.53	0.84	0.7	0.21	0.41	0.2	0.12	0.05	0.17	0.0
	nuSHAP vs. |SHAP|	0.31	0.5	0.84	0.69	0.19	0.42	0.19	0.17	0.08	0.43	0.0

Tool SHAP produces results of very poor quality!

NuSHAP vs. *SHAP – run times

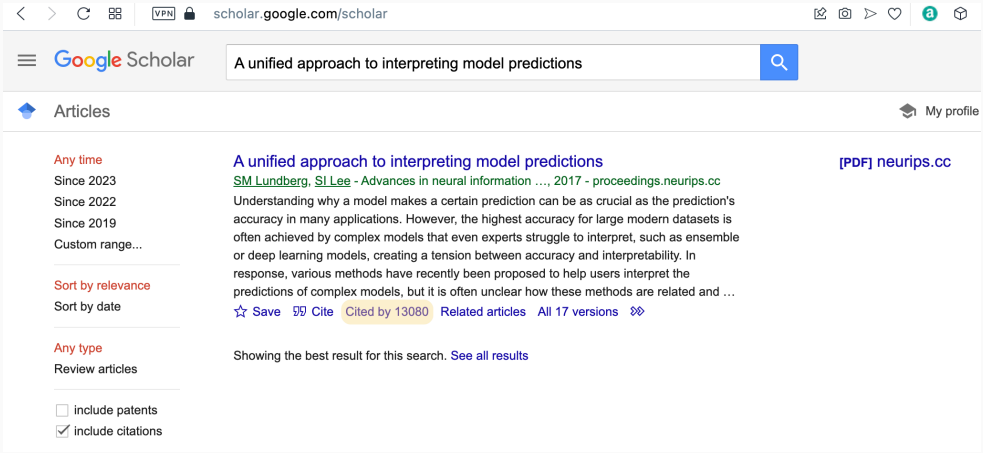
- Compute SHAP scores with NuSHAP and SHAP
- Compare features' rankings using Rank-Biased Overlap (RBO) [WMZ10]
  - Give weight to differences in first k elements of ranking
  - E.g. $\text{RBO}([0,1,2,3,4,5,6,7,8,9],[5,6,7,8,9,0,1,2,3,4],k=5)=0.0$
- Results for different datasets and ML models (using scikit, XGBoost):
  - LR: (adult, corral); DT: (iris, mux6); kNN: (connect_4, spambase, spectf); BT: (clean1, coil2000, dna); CNN: (MNIST)

	adult	corral	iris	mux6	connect_4	spambase	spectf	clean1	coil2000	dna	MNIST
SHAP	3.4	**0.1**	**0.0**	**0.0**	21.7	**0.5**	**0.7**	6.8	28.2	23.0	281.3
nuSHAP	**1.9**	1.5	1.5	1.5	**4.5**	2.7	2.9	**2.7**	**1.7**	**4.5**	**48.9**
#Samples	68045.5	63.7	955.4	63.9	9202.5	13654.1	36459.5	3929.6	2960.8	2756.8	2929.9

Outline

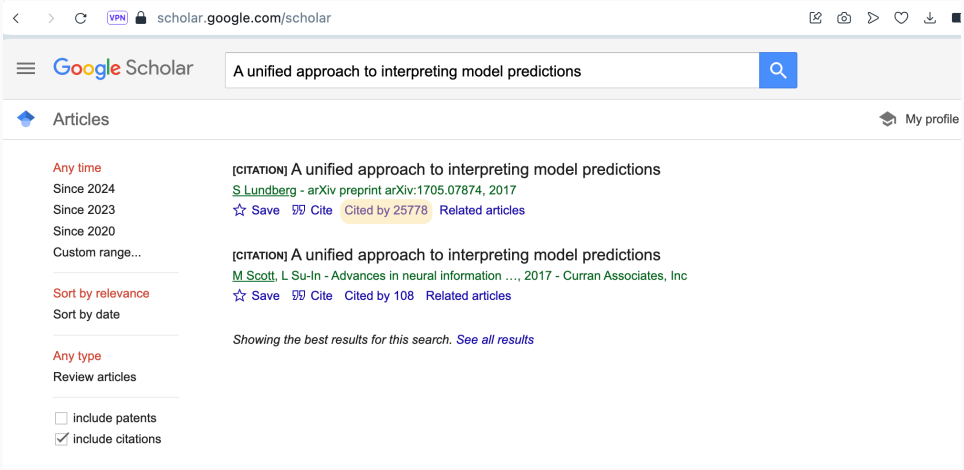
Can non-symbolic XAI's myths be stopped?

SHAP on 2023/05/31:



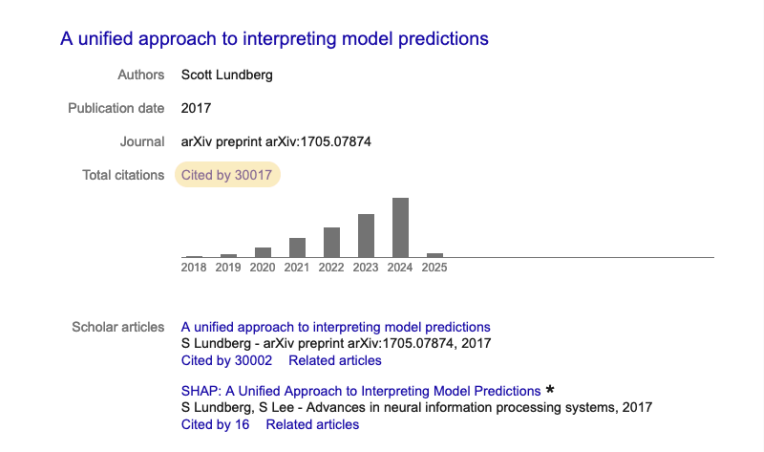
Can non-symbolic XAI's myths be stopped?

SHAP on 2024/09/15:



Can non-symbolic XAI's myths be stopped?

SHAP on 2025/01/12:

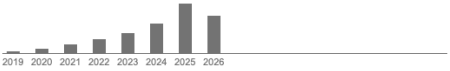


Can non-symbolic XAI's myths be stopped?

SHAP on 2026/07/16:

A unified approach to interpreting model predictions [PDF] from neurips.cc

Authors	Scott M Lundberg, Su-In Lee
Publication date	2017
Journal	Advances in neural information processing systems
Volume	30
Description	Understanding why a model makes a certain prediction can be as crucial as the prediction's accuracy in many applications. However, the highest accuracy for large modern datasets is often achieved by complex models that even experts struggle to interpret, such as ensemble or deep learning models, creating a tension between accuracy and interpretability. In response, various methods have recently been proposed to help users interpret the predictions of complex models, but it is often unclear how these methods are related and when one method is preferable over another. To address this problem, we present a unified framework for interpreting predictions, SHAP (SHapley Additive exPlanations). SHAP assigns each feature an importance value for a particular prediction. Its novel components include:(1) the identification of a new class of additive feature importance measures, and (2) theoretical results showing there is a unique solution in this class with a set of desirable properties. The new class unifies six existing methods, notable because several recent methods in the class lack the proposed desirable properties. Based on insights from this unification, we present new methods that show improved computational performance and/or better consistency with human intuition than previous approaches.
Total citations	Cited by 64411



Can non-symbolic XAI's myths be stopped?

SHAP on 2026/07/16:

But: theoretical SHAP scores can mislead & results of tool SHAP of poor quality!

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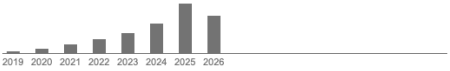
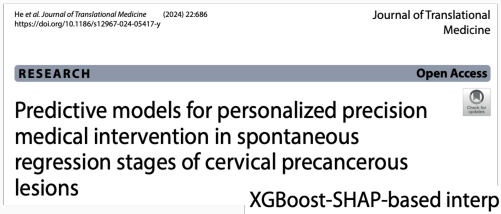


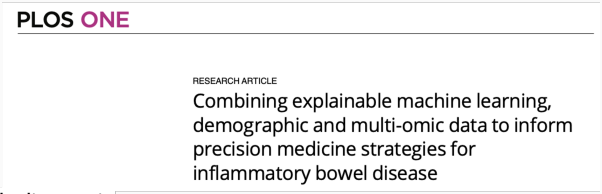
Many, many high-risk uses of SHAP & clones


He et al. Journal of Translational Medicine (2024) 22:686
https://doi.org/10.1186/s12967-024-05417-y

Journal of Translational Medicine

RESEARCH Open Access

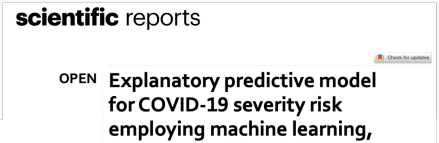
Predictive models for personalized precision medical intervention in spontaneous regression stages of cervical precancerous lesions


PLOS ONE

RESEARCH ARTICLE

Combining explainable machine learning, demographic and multi-omic data to inform precision medicine strategies for inflammatory bowel disease

XGBoost-SHAP-based interpretable diagnostic framework for alzheimer's disease

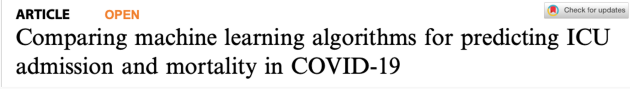

scientific reports

OPEN Explanatory predictive model for COVID-19 severity risk employing machine learning, shapley addition, and LIME

Hypothesis-free discovery of novel cancer predictors using machine learning

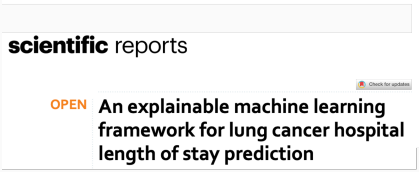
Explainable AI-based Deep-SHAP for mapping the multivariate relationships between regional neuroimaging biomarkers and cognition

npj | Digital Medicine

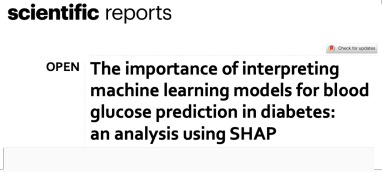

ARTICLE OPEN

Comparing machine learning algorithms for predicting ICU admission and mortality in COVID-19

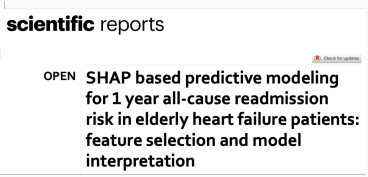
Interpretable prediction of 30-day mortality in patients with acute pancreatitis based on machine learning and SHAP


scientific reports

OPEN An explainable machine learning framework for lung cancer hospital length of stay prediction


scientific reports

OPEN The importance of interpreting machine learning models for blood glucose prediction in diabetes: an analysis using SHAP


scientific reports

OPEN SHAP based predictive modeling for 1 year all-cause readmission risk in elderly heart failure patients: feature selection and model interpretation

What next?

What next? Massive retraction (of ~~tens~~ hundreds of thousands) of papers??

thanks to ChatGPT...

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& beware of high-risk uses of SHAP! ☹

thanks to ChatGPT...

What’s the bottom line?

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- Use(r)s of non-symbolic XAI experience a persistent "*Don't Look Up*" moment...

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BTW, there are a multitude of proposed uses of LIME/SHAP in medicine... ⚠

Conclusions

- Brief contact with logic-based symbolic XAI
  - Rigorous definition of abductive/constrastive explanations
- Highlighted the flaws of Shapley values for XAI, i.e. the SHAP scores
- Detailed corrections to Shapley values for XAI
  - Right connection between logic-based XAI by feature selection and (corrected) XAI by feature attribution
- Overviewed nuSHAP
  - Still a prototype...
- Research directions?

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    - **Challenge accepted!**

Q & A

"Research is seeing what everybody else has seen and thinking what nobody else has thought."

(A. Szent-Gyorgyi)

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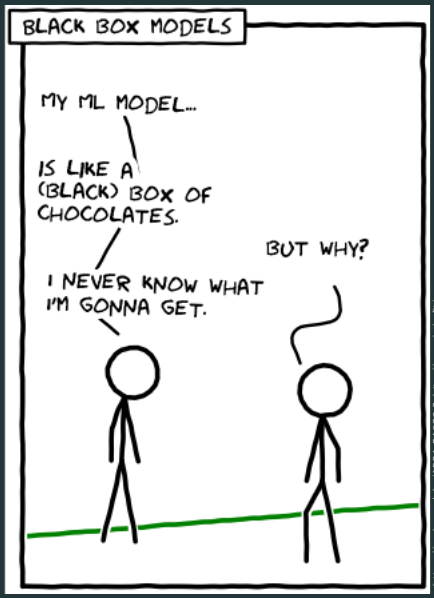
A. Ignatiev, X. Huang, Y. Izza, O. Létoffé, M. Cooper, N. Asher, N. Narodytska, J. Planes, A. Morgado, R. Bejar, C. Mencía, P. Stuckey, R. Passos, F. Chiariello, M. Siala, J. Lefebre-Lobaina, A. Hurault, H. Hu et al.
And: Gemini & ChatGPT


BLACK BOX MODELS
MY ML MODEL...
IS LIKE A (BLACK) BOX OF CHOCOLATES.
I NEVER KNOW WHAT I'M GONNA GET.
BUT WHY?
https://arxiv.org/abs/1901.01686 & http://cmx.io/edit/

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