Model Change for Description Logic Concepts

Ana Ozaki[1] , **Jandson S Ribeiro**[2]
[1]University of Oslo, Norway
[2]Cardiff University, UK
ana.ozaki@uib.no, ribeiroj@cardiff.ac.uk

Abstract

We consider the problem of modifying a description logic concept in light of models represented as pointed interpretations. We call this setting *model change*, and introduce a formal notion of revision and argue that it does not reduce to a simple combination of eviction (removing models) and reception (adding models), contrary to intuition. We provide positive and negative results on the compatibility of eviction and reception for $\mathcal{EL}_\bot$ and $\mathcal{ALC}$ description logic concepts and on the compatibility of revision for $\mathcal{ALC}$ concepts.

Keywords: Belief Change, Model Change, Description Logics, Pointed Models.

Statement and Information

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A copy of the paper can be accessed at https://arxiv.org/abs/2603.05562 We confirm that, although accepted at AAAI, the paper was not presented to a major dedicated KR audience. The paper has received an Outstanding Paper Award from AAAI26.

1 Introduction

Keeping beliefs updated is a central problem in knowledge representation that has been investigated in the context of different logics and applications. In the main streams of belief change research, the belief base is finite and the pieces of information expressing how to modify it are expressed as sets of formulae in the underlying logic (Hansson, 1999; Gärdenfors, 1988; Alchourrón, Gärdenfors, and Makinson, 1985). In many scenarios, however, using sets of models to specify the observed change is more suitable than using formulae. This is well studied in the context of *learning from interpretations* (De Raedt, 1997), where the goal is to find a concise formula that is consistent with models labelled as positive or negative. In the context of description logic (DL), the process of building an ontology usually goes through stages where the person creating it studies possible models of world, discarding models when they are proven false and adding new models previously not considered. The next examples illustrate model change operations for DL concepts.

Example 1. *Araci is visiting a zoo in Australia and knows little about Australian animals. She knows that a platypus is a mammal that lays eggs, so her belief on platypus is*

$$\mathsf{Platypus} \equiv \mathsf{Mammal} \sqcap (\exists \mathsf{lays}.\mathsf{Egg}).$$

She sees a platypus 'd' but knows nothing about their diet.

So, she entertains the two following possible worlds $(\mathcal{I}_1, d)$ *and* $(\mathcal{I}_2, d)$ *with* $\{d, e\} \subseteq \Delta^{\mathcal{I}_1} = \Delta^{\mathcal{I}_2}$*:*

$\mathcal{I}_1$:		$\mathcal{I}_2$:	
	$d \in \mathsf{Mammal}^{\mathcal{I}_1}$		$d \in \mathsf{Mammal}^{\mathcal{I}_2}$
	$d \notin \mathsf{Herbivore}^{\mathcal{I}_1}$		$d \in \mathsf{Herbivore}^{\mathcal{I}_2}$
	$(d, e) \in \mathsf{lays}^{\mathcal{I}_1}$		$(d, e) \in \mathsf{lays}^{\mathcal{I}_2}$
	$e \in \mathsf{Egg}^{\mathcal{I}_1}$		$e \in \mathsf{Egg}^{\mathcal{I}_2}$

She catches the platypus eating a small insect, which makes Araci retract the pointed interpretation $(\mathcal{I}_2, d)$*. So, she changes her conceptual beliefs about platypuses to*

$$\mathsf{Platypus} \equiv \mathsf{Mammal} \sqcap (\exists \mathsf{lays}.\mathsf{Egg}) \sqcap \neg \mathsf{Herbivore}.$$

Example 1 illustrates the process of removing a model from the concept, producing a concept closer to the real world. This change operation is called *eviction* (Guimarães, Ozaki, and Ribeiro, 2023), and should minimally modify the concept. In the paper, we have identified 3 main kinds of change operations where the input is a set of interpretations: eviction, which consists of only removing models; reception, which incorporates models; and revision, which combines removal with the incorporation of models in a single operation. We showed that revision does not reduce to a simple combination of eviction and reception, contrary to intuition. For each of these kinds of operations, we framed the precise set of rationality postulates and the class of constructive operations characterised by such postulates. In our framework, we require that the result be a finite base. Due to the expressiveness of the logics, it is not possible to define eviction, reception or revision in every logic. We address this problem under the name of **compatibility**: we show that several logics are not eviction, reception, and revision compatible. As a result, we investigate constraints general enough not to restrict much of the object language of the logic, or on the interpretations, and identify compatibility with reception, eviction, and revision.

2 Eviction and Reception: DL Concepts

Eviction and reception, respectively, refer to the processes of removing a set of models from a base and incorporating a set of models into a base. In some scenarios, not all sets of models need to be taken into account. For instance, some DLs have the finite model property or the tree-shaped property. So, it makes sense to also consider model change operators that only take into account such classes of models.

Definition 1. *A model change operator in a class $\mathcal{C}$ of models is a function $\circ : \mathcal{P}_f(\mathcal{L}) \times \mathcal{C} \rightarrow \mathcal{P}_f(\mathcal{L})$, mapping each finite base $\mathcal{B}$ into a finite base $\mathcal{B}'$ in light of a set of models.*

Here we investigate eviction and reception on DL concepts, focusing on the prototypical DLs $\mathcal{ALC}$ and $\mathcal{EL}$. In the following, we denote by $\Lambda(\mathcal{EL}_{\perp\text{concepts}})$ and $\Lambda(\mathcal{ALC}_{\text{concepts}})$ the satisfaction systems for $\mathcal{EL}_\perp$ and $\mathcal{ALC}$ concepts with pointed interpretations as models. Table 1 summarises our results.

Sat. System	Eviction	Reception
$\Lambda(\mathcal{EL}_{\perp con.})$	yes	no
$\Lambda(\mathcal{EL}_{\perp con.})^\dagger$	yes	yes
$\Lambda(\mathcal{ALC}_{con.})$	no	no
$\Lambda(\mathcal{ALC}_{con.})^\ddagger$	yes	yes

Table 1: Eviction and reception-compatibility for DL concepts. † is for the case pointed interpretations can only be tree-shaped and ‡ is for the case they can only be tree-shaped, over finite signatures, and sets of models can only be finite.

3 Revision

Model revision incorporates a set of models while also guaranteeing that another set of models is removed. For instance, in Example 2, below $(\mathcal{I}_2, d_2)$ had to be removed, while $(\mathcal{I}_1, d_1)$ had to be added. Revision cannot be defined by assembling reception and eviction, as reception can add models required to be evicted and vice versa.

Example 2. *Let $\mathcal{B} = \{\exists r.\top\}$ be an $\mathcal{EL}_\perp$ concept on the signature $\mathsf{N_C} = \{A\}$ and $\mathsf{N_R} = \{r\}$. Let $(\mathcal{I}_1, d_1)$ and $(\mathcal{I}_2, d_1)$ be pointed models with $\Delta^{\mathcal{I}_1} = \{d_1, d_2\}, \Delta^{\mathcal{I}_2} = \{d_1, d_2, d_3\}$ and*

$$\mathcal{I}_1 : \boxed{A^{\mathcal{I}_1} = \{d_2\}, r^{\mathcal{I}_1} = \{(d_1, d_2)\}}$$

$$\mathcal{I}_2 : \boxed{A^{\mathcal{I}_2} = \{d_1\}, r^{\mathcal{I}_2} = \{(d_1, d_2), (d_2, d_3)\}}$$

We want to revise $\mathcal{B}$ with $(\{(\mathcal{I}_1, d_1)\}, \{(\mathcal{I}_2, d_1)\})$, that is, receive $(\mathcal{I}_1, d_1)$ and evict $(\mathcal{I}_2, d_1)$. Combining rational eviction with rational reception, in any order, is not strong enough to achieve revision. A rational eviction of $\mathcal{B}$ with $(\mathcal{I}_2, d_1)$ is $\mathcal{B}' = \{\exists r^3.\top\}$. However, incorporating $(\mathcal{I}_1, d_1)$ to it yields the base $\{\exists r.\top\}$ again, which contains $(\mathcal{I}_2, d_1)$. On the other hand, reception of $\mathcal{B}$ with $(\mathcal{I}_1, d_1)$ does not change $\mathcal{B}$, as $(\mathcal{I}_1, d_1)$ is a model of $\exists r.\top$. Eviction of $\mathcal{B}$ with $(\mathcal{I}_2, d_1)$ gives $\exists r^3.\top$, which does not contain $(\mathcal{I}_1, d_1)$.

Here, we briefly consider the revision of DL concepts. Although it is not possible to define revision as a combination of eviction and reception, we have shown that revision compatibility is closely related to both eviction and reception. In fact, eviction and reception are special cases of revision, whereas revision can only be performed in classes of models compatible with both reception and eviction. This connection between revision with eviction and reception allows to translate (in)compatibility results from eviction and reception to revision. Based on this major result, and the results on Table 1 we showed that neither $\Lambda(\mathcal{EL}_{\perp\text{concepts}})$ nor $\Lambda(\mathcal{ALC}_{\text{concepts}})$ are, in general, revision-compatible. We establish that these satisfaction systems are also not revision-compatible when we restrict to finite tree-shaped pointed interpretations.

Theorem 1. *$\Lambda(\mathcal{EL}_{\perp concepts})$ and $\Lambda(\mathcal{ALC}_{concepts})$ are not revision-compatible in the binary class of finite tree-shaped pointed interpretations. This also holds if we restrict to finite sets of models and if we restrict to a finite signature.*

We consider the binary class of finite tree-shaped pointed interpretations for finite sets of models *union their closure under bisimulation* over a finite signature. This class of revision-compatible.

Theorem 2. *$\Lambda(\mathcal{ALC}_{concepts})$ is revision-compatible in the binary class of sets of pointed interpretations which are the closure under bisimulation of finite sets of finite tree-shaped pointed interpretations over a (unique) finite signature.*

4 Conclusion

We investigated eviction and reception for DL concepts, establishing various results considering different classes of models. We find classes of models where we obtain eviction and reception compatibility for both $\mathcal{ALC}$ and $\mathcal{EL}_\perp$ concepts. It turns out that the class of models where we obtain positive results for $\mathcal{ALC}$ is much more restricted than the class for $\mathcal{EL}_\perp$. We also introduce the notion of model revision and relate various postulates with the revision operation. Revision cannot be seen as a mere combination of eviction and reception, which is evidenced by negative results for DL concepts.

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