An Extension-Based Argument-Ranking Semantics: Social Rankings in Abstract Argumentation (Extended Abstract)

Lars Bengel[1], **Giovanni Buraglio**[2], **Jan Maly**[2,3], **Kenneth Skiba**[1]
[1]Artificial Intelligence Group, University of Hagen, Hagen, Germany
[2]Institute of Logic and Computation, TU Wien, Wien, Austria
[3]Institute of Data, Process and Knowledge Management, Vienna University of Economics and Business, Wien, Austria
{lars.bengel, kenneth.skiba}@fernuni-hagen.de, giovanni.buraglio@tuwien.ac.at, jan.maly@tuwien.ac.at

Abstract

In this paper, we introduce a new family of argument-ranking semantics which can be seen as a refinement of the classification of arguments into skeptically accepted, credulously accepted and rejected. To this end we use so-called social ranking functions which have been developed recently to rank individuals based on their performance in groups. We provide necessary and sufficient conditions for a social ranking function to give rise to an argument-ranking semantics satisfying the desired refinement property.

1 Introduction

Classifying whether an argument is accepted or rejected in the context of a larger discussion is one of the core problems of computational models of argumentation. In abstract argumentation, this is usually achieved by checking if an argument is contained in a set of jointly acceptable arguments, called *extensions*, according to one of several well-established semantics. While these semantics provide a natural way to rank arguments based on the larger context of the debate at hand, they only allow us to distinguish three types of arguments: the ones that are *skeptically accepted*, i. e. that are contained in every extension; the ones that are *credulously accepted*, i. e. that are contained in at least one extension; and the ones that are *rejected*, i. e. that are not contained in any extension. However, this classification is too restrictive (Amgoud and Ben-Naim 2013).

In the paper (Bengel et al. 2025), we present a true refinement of the classification in skeptically, credulously and rejected arguments, by combining two strands of literature, namely *extension-ranking semantics* (Skiba et al. 2021) and *social ranking functions* (Moretti and Öztürk 2017). For an abstract argumentation framework (Dung 1995), we compute a preorder over the powerset of arguments (*extension ranking*) and then apply *social ranking functions* to receive an argument ranking that refines the classification in skeptically, credulously and rejected arguments. Meaning that the skeptically accepted arguments are ranked better than credulously accepted arguments and those are ranked better than rejected arguments, and within each of these groupings the arguments are also ranked. In addition, we show which axiomatic properties are sufficient and necessary for a social ranking function to give rise to such a ranking.

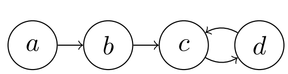

Figure 1: Abstract argumentation framework F_1 from Example 1.

2 Preliminaries

An *abstract argumentation framework* (AF) (Dung 1995) is a directed graph $F = (A, R)$ where A is a finite set of *arguments* and R is an *attack relation* $R \subseteq A \times A$. An argument a is said to *attack* an argument b if $(a, b) \in R$. We say that an argument a is *defended by a set* $E \subseteq A$ if every argument $b \in A$ that attacks a is attacked by some $c \in E$.

A set $E \subseteq A$ is a *conflict-free* (cf) set iff $\forall a, b \in E$, $(a, b) \notin R$; *admissible* (ad) set iff it is *conflict-free* and it defends its elements; *complete* (co) set iff it is *admissible* and contains every argument it defends. We refer to these sets as *extension-based semantics* σ. With extension-based semantics, we can define the acceptance status of any argument, namely *skeptically accepted* (belonging to each σ-set), *credulously accepted* (belonging to some σ-set) and *rejected* (belonging to no σ-set). Given an AF F and an extension-based semantics σ, we use (respectively) $sk_\sigma(F)$, $cred_\sigma(F)$ and $rej_\sigma(F)$ to denote these sets of arguments.

Example 1. *Consider the AF $F = (A, R)$ depicted as a directed graph in Figure 1.*

We see that F_1 has three complete sets $\{a\}, \{a, c\}$ and $\{a, d\}$. In addition, $a \in sk_{co}(F_1)$, $c, d \in cred_{co}(F_1)$, and $b \in rej_{co}(F_1)$.

An *extension-ranking semantics* (Skiba et al. 2021) maps an AF to a preorder over the powerset of argument $\mathcal{P}(A)$. These semantics are used to formalise whether a set E is more plausible to be accepted than another set E'. Skiba et al. (2021) have introduced extension-ranking semantics focusing on generalising Dung's extension-based semantics. Due to space restriction we refer the reader to Skiba et al. (2021) for more information.

A *social ranking function* (Moretti and Öztürk 2017) ξ

maps a preorder $\sqsupseteq$ on $\mathcal{P}(A)$ to a preorder on A. The most prominent social ranking function is the *lexicographic excellence operator* (lex-cel) (Bernardi, Lucchetti, and Moretti 2019). It ranks elements based on the best sets they appear in, proceeding lexicographically if there are ties.

3 Refining Acceptance of Arguments

Next, we combine extension-ranking semantics and social ranking functions to refine the acceptance status of arguments. Given an AF $F = (A, R)$, we first compute an extension ranking with respect to a given extension-ranking semantics. Then, the resulting preorder on $\mathcal{P}(A)$ is used as an input for a social ranking function, to obtain a preorder on A. Due to their wide domain of applicability, social ranking functions have received considerable attention in recent years. Consequently, a number of different functions can be found in the literature. (Algaba et al. 2021; Bernardi, Lucchetti, and Moretti 2019; Haret et al. 2018; Khani, Moretti, and Öztürk 2019). Hence, to understand what constitutes a good social ranking function in our context, we define a general semantics using social ranking solutions with respect to an extension ranking. An argument a is at least as strong as argument b if the social ranking function ξ applied to the extension ranking τ returns that a is at least as strong as b.

Example 2. *Consider AF F_1 from Example 1. Argument a is contained in all three complete sets, while c and d are part of one complete set each. Argument b is rejected. Consequently, $a \succeq^{\textit{lex-cel}_{\textit{r-co}}} c$ and $a \succeq^{\textit{lex-cel}_{\textit{r-co}}} d$. To compare c and d further, we use the* complete extension-ranking semantics r-co *(Skiba et al. 2021). The set $\{d\}$ is more plausible to accept with respect to* r-co *than any set containing c (except $\{a, c\}$), thus $d \succeq^{\textit{lex-cel}_{\textit{r-co}}} c$. The entire ranking is:*

$$a \succ_{F_1}^{\textit{lex-cel}_{\textit{r-co}}} d \succ_{F_1}^{\textit{lex-cel}_{\textit{r-co}}} c \succ_{F_1}^{\textit{lex-cel}_{\textit{r-co}}} b$$

In Example 2, the skeptically accepted argument a is ranked above the other arguments and the rejected argument b is ranked below all other arguments. In between, there are the two credulous accepted arguments c and d. With our proposed approach, we can further compare c and d, by stating that d is a stronger argument than c. For skeptically or rejected arguments, we can also further compare them within their own class by using the underlying extension-ranking semantics. Thus, $\succeq^{\text{lex-cel}_{\text{r-co}}}$ is a true refinement of the acceptance classification of arguments with respect to the complete extension semantics. Besides complete extension semantics, our approach covers all of Dung's extension semantics (Dung 1995).

More generally, we show which axiomatic properties are sufficient and necessary for a social ranking function and an extension-ranking semantics to ensure that any Social ranking argument-ranking semantics induces a ranking that truly refines the acceptance status of arguments. In the admissible case, the extension-ranking semantics has to generalise admissibility. For the social ranking function, it has to hold that the worst ranked sets according to the underlying extension-ranking can be further subdivided without changing any strict preferences. Moreover, if every set containing argument a is more plausible to accept than every set containing b, then a should be ranked better than b.

4 Conclusion

We have combined well-known approaches from abstract argumentation with social ranking functions to define a new family of argument-ranking semantics. The resulting semantics are generalisations of the acceptance classifications for abstract argumentation. Thus, the skeptically accepted arguments are ranked better than credulously accepted arguments and those are ranked better than rejected arguments, and within each of these groupings the arguments are also ranked. All the methods used are off-the-shelf approaches and already discussed in the literature, showing the connection between social ranking function and argumentation as well as the simplicity of this approach.

References

Algaba, E.; Moretti, S.; Rémila, E.; and Solal, P. 2021. Lexicographic solutions for coalitional rankings. *Social Choice and Welfare* 57(4):817–849.

Amgoud, L., and Ben-Naim, J. 2013. Ranking-based semantics for argumentation frameworks. In *Scalable Uncertainty Management - 7th International Conference, SUM 2013*, 134–147. Springer.

Bengel, L.; Buraglio, G.; Maly, J.; and Skiba, K. 2025. An extension-based argument-ranking semantics: Social rankings in abstract argumentation. In Walsh, T.; Shah, J.; and Kolter, Z., eds., *Proceedings of the Thirty-ninth AAAI Conference on Artificial Intelligence, AAAI 2025, February 25 - March 4, 2025, USA*, 14790–14797. AAAI Press.

Bernardi, G.; Lucchetti, R.; and Moretti, S. 2019. Ranking objects from a preference relation over their subsets. *Social Choice and Welfare* 52(4):589–606.

Dung, P. M. 1995. On the Acceptability of Arguments and its Fundamental Role in Nonmonotonic Reasoning, Logic Programming and n-Person Games. *Artificial Intelligence*.

Haret, A.; Khani, H.; Moretti, S.; and Öztürk, M. 2018. Ceteris paribus majority for social ranking. In *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence, IJCAI 2018*, 303–309. ijcai.org.

Khani, H.; Moretti, S.; and Öztürk, M. 2019. An ordinal banzhaf index for social ranking. In *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI 2019*, 378–384. ijcai.org.

Moretti, S., and Öztürk, M. 2017. Some axiomatic and algorithmic perspectives on the social ranking problem. In *Algorithmic Decision Theory - 5th International Conference, ADT 2017*, 166–181. Springer.

Skiba, K.; Rienstra, T.; Thimm, M.; Heyninck, J.; and Kern-Isberner, G. 2021. Ranking extensions in abstract argumentation. In *Proceedings of the Thirtieth International Joint Conference on Artificial Intelligence, IJCAI 2021*, 2047–2053. ijcai.org.