A Paraconsistency Framework for Inconsistency Handling in Qualitative Spatial and Temporal Reasoning

Yakoub Salhi[1] , **Michael Sioutis**[2]
[1]CRIL UMR 8188, Université d'Artois & CNRS, France
[2]LIRMM UMR 5506, Université de Montpellier & CNRS, France
salhi@cril.fr, msioutis@lirmm.fr

Abstract

Inconsistency handling is a fundamental problem in knowledge representation and reasoning. In this paper, we study this problem in the context of qualitative spatio-temporal reasoning, a framework for reasoning about space and time in a symbolic, human-like manner, by following an approach similar to that used for defining paraconsistent logics; paraconsistency allows deriving informative conclusions from inconsistent knowledge bases by mainly avoiding the principle of explosion. Inspired by paraconsistent logics, such as Priest's logic LPm, in (Salhi and Sioutis 2023b) we introduce the notion of paraconsistent scenario (i.e., a qualitative solution), which can be seen as a scenario that allows a conjunction of base relations between two variables, e.g., x *precedes* $\wedge$ *follows* y. Further, we present several interesting theoretical properties that concern paraconsistent scenarios, including computational complexity results, and describe two distinct approaches for computing paraconsistent scenarios and solving other related problems. Moreover, we provide implementations of our two methods for computing paraconsistent scenarios and experimentally evaluate them using different strategies/metrics. Finally, we show that our paraconsistent scenario notion allows us to adapt to qualitative reasoning one of the well-known inconsistency measures employed in the propositional case, namely, contension measure.

Introduction

Inconsistency may arise for many reasons: human error, multi-source information, imprecision and vagueness, noisy data, information evolution over time, etc. This explains the need for inconsistency-tolerant systems to deal with real-world situations. The Knowledge Representation & Reasoning community has extensively studied this topic leading to several inconsistency handling works, e.g., (Rescher and Manor 1970; Reiter 1980; Benferhat, Dubois, and Prade 1995; Besnard and Hunter 2008; Tanaka et al. 2013; Potyka and Thimm 2015; Salhi and Sioutis 2023a). In this work, we are interested in the use of paraconsistency for inconsistency handling in Qualitative Spatio-Temporal Reasoning (to be introduced in the sequel). A logic is paraconsistent if it does not validate the principle of explosion, which states that any formula can be proven from contradiction. In particular, Priest's minimally inconsistent logic of paradox LPm (Priest 1991) avoids this principle by allowing variables to be both true and false. This can be seen as a way to allow for the existence of contradictions without collapsing into triviality: consistent and inconsistent elements can coexist in a logical statement without rejecting it as *false*.

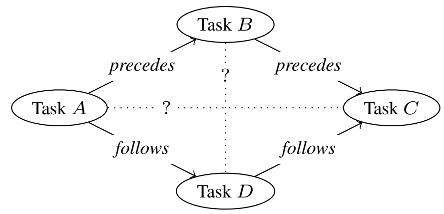

Figure 1: A simplified temporal constraint network, where ? denotes the *disjunction* of all base relations; the constraint between Tasks B and D (and, equivalently, A and C) is not repairable, but we can replace ? with the *conjunction* '*precedes* $\wedge$ *follows*' and achieve paraconsistency.

In everyday natural language descriptions, one typically uses expressions such as *inside* or *during* to spatially or temporally relate one object with another object or oneself, without providing the exact metric information about these entities. An AI framework that aims to capture this type of human-like representation and reasoning pertaining to space and time is known as Qualitative Spatio-Temporal Reasoning (QSTR) (Dylla et al. 2017; Ligozat 2013). Specifically, QSTR is a major field of study in Knowledge Representation and Reasoning that deals with the concepts of space and time in an abstract, natural manner, with applications in many areas such as visual sensemaking (Suchan, Bhatt, and Varadarajan 2021) and qualitative case-based reasoning and learning (Homem et al. 2020), to name some recent ones. More formally, QSTR restricts the vocabulary of rich mathematical theories that deal with spatial and temporal entities to simple qualitative constraint languages, which can be used to form interpretable spatio-temporal constraint networks of disjunctions of base relations, such as the one shown in Figure 1. However, as with any other typical logic- or constraint-based framework, QSTR is not immune to contradictions that may be present in information.

Motivation

Naturally, the motivation behind studying paraconsistency in the context of QSTR in (Salhi and Sioutis 2023b) draws from the rich literature in paraconsistency itself. However, we present an example here to help the reader understand one of many cases of how this notion can apply to QSTR. Consider Figure 1, and let us ground it in a realistic scenario of task scheduling in a factory. We can view Task D as an *inspection* task of a product in the production pipeline, and the other tasks as necessary components in the manufacturing process of that product. Clearly, a mistake occured in the design of the pipeline, as the schedule is unfeasible. The constraint between Tasks B and D is not repairable, so, to restore consistency, we would have to repair some other constraints. However, this may be impossible too, due to hard dependencies in the pipeline, e.g., product preparation, say Task B, *precedes* product packaging, say Task C. Instead of rejecting the entire schedule, we opt to acknowledge the contradiction and reason with it. Here, we can retrieve a paraconsistent configuration where Task B both *precedes and follows* Task D. This not only helps us to understand the contradiction, but, in this example, to also observe that the *inspection* task was probably meant to occur both at an earlier and at a later stage of the production pipeline and should thus be replicated.

Contributions

We summarize our theoretical and practical contributions in (Salhi and Sioutis 2023b) as follows.

Our *first* contribution is the introduction of the notion of paraconsistent scenario (para-scenario for short). To some extent, it can be seen as an adaptation of the approach used for defining LPm to QSTR. Indeed, similarly to LPm, where an interpretation can assign a propositional variable more than one truth value, our base idea consists in allowing constraints to involve a conjunction of more than one base relation, as a means to achieve compatibility with other constraints (see Figure 1); then, we focus on the para-scenarios that are as consistent as possible, which are obtained by avoiding as much as possible the use of such conjunctions.

Our *second* contribution is the theoretical study of several interesting properties of para-scenarios. In particular, we show that the problem of determining whether an interpretation is a para-scenario is coNP-complete in the case of several well-known QSTR formalisms. In sum, this theoretical result is mainly based on the complement problem of 3-coloring.

Our *third* contribution involves providing and evaluating two open-source approaches for solving the problem of para-scenario computation and other related problems. The first approach is based on a notion of constraint freezing (cf. (Condotta, Ligozat, and Saade 2007)) within calls to a native qualitative reasoner: when a constraint is frozen, it cannot lose any base relation during solving, but, in contrast to (Condotta, Ligozat, and Saade 2007), it can participate in compositions with other constraints. The second approach consists of using SAT-based encodings, where we involve the problems of X-minimal model computation and Partial MaxSAT in particular.

Finally, our *fourth* contribution is showing how the notion of para-scenario can be used for inconsistency measurement. Indeed, we propose inconsistency measures that can be seen as the first adaptation of the well-known contension measure to QSTR (Grant and Hunter 2011). This contribution is provided just as a concrete example of how our framework can be exploited to analyze/measure inconsistency. The definition of such measures in the literature is commonly guided by rationality postulates. In our study, we show that our measures fulfill postulates that enjoy a broad consensus.

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