A Kinematics Principle For Iterated Revision (Extended Abstract)

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Abstract

We propose a qualitative kinematics principle for iterated revision of epistemic states represented by total preorders. As new information, we allow sets of conditional beliefs, going far beyond the current state of the art of belief revision. We introduce a qualitative conditioning operator for total preorders which is compatible with conditioning for Spohn's ranking functions, and transfer the technique of c-revisions to total preorders to provide a proof of concept for our kinematics principle at least for special revision scenarios.

1 Introduction

In probabilistics, the *kinematics principle* claims that when the probability of a fact is changed, the conditional probabilities given this fact should be preserved. This principle is implemented by Jeffrey's rule, and it is one of the basic principles underlying Bayesian networks (Pearl 1988). Less well known, the probabilistic idea of kinematics has been significantly extended by the property of *subset independence* (Shore and Johnson 1980). That work had mainly inspired the work in (Kern-Isberner, Sezgin, and Beierle 2023) which we briefly summarize in this extended abstract.

Transferred to qualitative environments where epistemic states are represented by total preorders, subset independence deals with the following epistemic revision problem where the new information can be split over cases: Let Ψ be an epistemic state represented as a total preorder $\preceq_\Psi$ on its possible worlds Ω. Let $A_1, \ldots, A_n$ be exhaustive and exclusive propositions (cases), and let $\Delta = \Delta_1 \cup \ldots \cup \Delta_n$ be a set of conditionals, with subsets Δ_i the premises of which imply A_i, and let $S = \bigvee_{j \in J} A_j$ with $\emptyset \neq J \subseteq \{1, \ldots, n\}$. How should Ψ be revised by Δ and S in a rational way to yield a posterior state $\Psi^\bullet = \Psi \bullet (\Delta \cup \{S\})$ such that conditional beliefs in Ψ and Δ are treated adequately, and such that conditional revised beliefs given A_i are unaffected by the information provided by S?

The A_i's represent different cases, and the Δ_i's provide new information referring (only) to the respective cases. Moreover, the proposition S expresses a belief which of the cases might be plausible. Transferring the ideas of kinematics to this qualitative revision problem, a qualitative kinematics principle for revision operators $\bullet$ of total preorders can be formulated as follows:

$$\textbf{(QK)} \quad (\Psi \bullet (\Delta \cup \{S\}))|A_i = (\Psi|A_i) \bullet \Delta_i \qquad (1)$$

(QK) has two crucial implications concerning the relevance of complex information under revision: first, the plausibility of cases (expressed by S) does not affect the conditional beliefs for each case A_i, and second, for the posterior conditional beliefs given A_i only the respective new information Δ_i is relevant. Note that in (1), the Δ_i's may be empty, so that exhaustiveness of the cases is not mandatory, and that $S = \bigvee_{1 \leq j \leq n} A_j$, i.e., S is a tautology, is possible. In the latter case, when we further impose that tautologies should have no effect on the revision result, i.e., $\Psi \bullet (\Delta \cup \{\top\}) = \Psi \bullet \Delta$, kinematics postulates the commutativity of conditionalization and revision on each A_i:

$$(\Psi \bullet \Delta)|A_i = (\Psi|A_i) \bullet \Delta_i \qquad (2)$$

Kinematics may significantly help reducing the complexity of belief revision operations. Beyond these technical advantages, kinematics also implements a notion of relevance and local reasoning that make the outcomes of intelligent systems more intelligible and understandable for humans.

In (Kern-Isberner, Sezgin, and Beierle 2023), we introduce a qualitative conditionalization for total preorders. Furthermore, we propose a transformation schema between ranking functions and total preorders that is compatible with crucial characteristics of conditionalization on both sides. With this schema, we transfer the technique of c-revisions (Kern-Isberner 2004) to total preorders so that qualitative kinematics (QK) can be satisfied at least in special cases.

2 Logical Background

We are working in a propositional environment with a propositional language $\mathcal{L}$, finitely generated from an alphabet Σ, with the usual connectives and notations. The set of all propositional interpretations over Σ is denoted by Ω. We call formulas $A_i \in \mathcal{L}$ $(i = 1, \ldots, n)$ *exclusive* iff $A_i A_j \equiv \bot$ for $i \neq j$, and *exhaustive* iff $A_1 \vee \ldots \vee A_n \equiv \top$.

We consider conditionals $(B|A)$ with $A, B \in \mathcal{L}$, where $(B|A)$ expresses *"If A then plausibly B"*. A *conditional belief base* Δ is a finite set of conditionals. Conditionals can be interpreted conveniently via total preorders (TPO), i.e., transitive and reflexive total relations on Ω. We focus on epistemic states Ψ which are represented by TPOs, i.e., $\Psi = (\Omega, \preceq_\Psi)$. The preorder $\preceq_\Psi$ is lifted to a relation between propositions in the usual way: $A \preceq_\Psi B$ if there is $\omega \models A$

such that $\omega \preceq_\Psi \omega'$ for all $\omega' \models B$. A conditional $(B|A)$ is accepted in Ψ, denoted by $\Psi \models (B|A)$, if $AB \prec_\Psi A\overline{B}$.

Ordinal Conditional Functions (OCF, also called *ranking functions*), are functions $\kappa : \Omega \to \mathbb{N} \cup \{\infty\}$ with $\kappa^{-1}(0) \neq \emptyset$ (Spohn 1988). For propositions $A \in \mathcal{L}$, we have $\kappa(A) := \min\{\kappa(\omega)|\omega \models A\}$. Conditionals are accepted by κ, written as $\kappa \models (B|A)$, if $\kappa(AB) < \kappa(A\overline{B})$. OCFs can be conditionalized by propositions A via $\kappa|A\,(\omega) = \kappa(\omega) - \kappa(A)$ for $\omega \models A$ (Spohn 1988).

3 Strategic C-revisions and Transformations

C-revisions are a general methodology to revise ranking functions with sets of (plausible) facts and conditionals.

Definition 1 (C-revisions for OCFs (Kern-Isberner 2004))**.** *Let κ be an OCF and $\Delta = \{(B_1|X_1), \ldots, (B_m|X_m)\}$ a set of conditionals. Then a* c-revision of κ by Δ *is an OCF $\kappa^* = \kappa * \Delta$ constructed from nonnegative integers η_i assigned to each $(B_i|X_i)$ and an integer κ_0 such that $\kappa^* \models \Delta$ and is given by $\kappa^*(\omega) = \kappa_0 + \kappa(\omega) + \sum_{\substack{1 \leqslant i \leqslant m \\ \omega \models X_i\overline{B}_i}} \eta_i$.*

The condition $\kappa^* \models \Delta$ induces a constraint system over the η_i, and with a *selection strategy* $\sigma : (\kappa, \Delta) \mapsto \vec{\eta}$, we can choose a specific solution vector $\vec{\eta}$ for each revision problem. If $\sigma(\kappa, \Delta) = \vec{\eta}$, the *c-revision of κ by Δ determined by* σ is $\kappa^*_{\vec{\eta}}$, denoted by $\kappa *_\sigma \Delta$, and $*_\sigma$ is a *strategic c-revision.*

Presupposing a special invariance property for selection strategies, it was shown in (Sezgin, Kern-Isberner, and Beierle 2021) that strategic c-revisions $*_\sigma$ satisfy *Generalized Ranking Kinematics (GRK)*, i.e., $(\kappa * (\Delta \cup \{S\}))|A_i = (\,\kappa|A_i\,) * \Delta_i$. We use this result to come up with an approach to revisions of TPOs by sets of conditionals that comply with (QK). For this, we need to define transformations between OCFs and TPOs which respect conditionalization. First, we define conditionalization for total preorders which shares important characteristics with Spohn's conditionalization for ranking functions.

Definition 2 (Conditionalization of total preorders)**.** *Let $\preceq_\Psi$ be a total preorder on Ω, let A be a propositional formula. The* conditionalization of Ψ on A*, denoted by $\Psi|A$, is defined as $\Psi|A = (Mod(A), \preceq_{\Psi|A})$ such that*

$$\omega_1 \preceq_{\Psi|A} \omega_2 \ \textit{iff} \ \omega_1 \preceq_\Psi \omega_2 \ \textit{for} \ \omega_1, \omega_2 \models A.$$

Regarding OCF$\leftrightarrow$TPO transformations, it is clear that each OCF κ induces uniquely a TPO and hence (the representation of) an epistemic state $\Psi_\kappa = (\Omega, \preceq_{\Psi_\kappa})$ by defining a transformation $\tau : \kappa \mapsto \Psi_\kappa$ via $\omega_1 \preceq_{\Psi_\kappa} \omega_2$ iff $\kappa(\omega_1) \leqslant \kappa(\omega_2)$. Conversely, for each TPO, there are infinitely many (equivalent) OCFs which induce that TPO. To make the transformation unique, we choose here the minimal one. Formally, we define a *transformation operator* $\rho : \Psi \mapsto \kappa_\Psi$ that takes an epistemic state $\Psi = (\Omega, \preceq_\Psi)$ and returns an OCF via $\kappa_\Psi(\omega) = \min_{\kappa \in \tau^{-1}(\Psi)} \{\kappa(\omega)\}$. These transformations are fully compatible with (TPO and OCF) conditionalization by consistent formulas A, since we have $\tau(\kappa)|A = \Psi_\kappa|A = \Psi_{(\,\kappa|A\,)} = \tau(\,\kappa|A\,)$ and $\tau(\,\kappa_\Psi|A\,) = \tau(\kappa_{\Psi|A})$. This yields in particular $\tau(\,\rho(\Psi)|A\,) = \tau(\rho(\,\Psi|A\,)) = \Psi|A$. We are now ready to transfer the approach of (strategic) c-revisions to total preorders.

4 Qualitative C-revisions and Kinematics

Let $\Psi = (\Omega, \preceq_\Psi)$ be an epistemic state with $\rho(\Psi) = \kappa_\Psi$ as the corresponding OCF, and let Δ be a set of conditionals. Let $*$ be a c-revision operator according to Def. 1, and let σ be a selection strategy for c-revisions, inducing a strategic c-revision operator $*_\sigma$. A *qualitative (strategic) c-revision* $\bullet_{(\sigma)}$ for Ψ is defined by setting $\Psi \bullet_{(\sigma)} \Delta = \tau(\kappa_\Psi *_{(\sigma)} \Delta)$.

In Theorem 15 of (Kern-Isberner, Sezgin, and Beierle 2023), we show that qualitative strategic c-revisions that employ a suitable strategy fulfill (QK) at least in special cases. In the following, we illustrate how kinematics can help reduce the complexity of revision by an example.

Example 1. *Let the epistemic state Ψ be specified by the following total preorder $\preceq_\Psi$ on the worlds of the language $\mathcal{L}$ over the signature $\Sigma = \{a, b, c, d\}$:*

$$abc\dot{d}, \overline{a}bcd, \overline{a}b\overline{c}\overline{d}, \overline{a}\overline{b}\overline{c}\dot{d} \prec_\Psi ab\overline{c}\dot{d}, a\overline{b}\dot{c}\dot{d}, \overline{a}\overline{b}\overline{c}d, \overline{a}bc\overline{d}, \overline{a}\overline{b}c\dot{d},$$

where overlining indicates negation, and dotted atoms can be positive or negative. We find that $\Psi \models (d|\overline{a}bc)$ because $\overline{a}bcd \prec_\Psi \overline{a}bc\overline{d}$, and $\Psi \models (\overline{c}|\overline{a}\overline{b})$ because $\overline{a}\overline{b}\overline{c} \prec_\Psi \overline{a}\overline{b}c$.

Now we want to revise Ψ by $\Delta = \{(\overline{d}|\overline{a}bc), (c|\overline{a}\overline{b})\}$. The premises $A_1 = \overline{a}bc, A_2 = \overline{a}\overline{b}$ are exclusive, and we split Δ into $\Delta_1 = \{(\overline{d}|\overline{a}bc)\}$ and $\Delta_2 = \{(c|\overline{a}\overline{b})\}$, leaving Δ_3 regarding the premise $A_3 = \neg(A_1 \vee A_2) \equiv a \vee \overline{a}b\overline{c}$ empty. Kinematics in the form (2) then claims that $(\Psi \bullet \Delta)|A_i = (\,\Psi|A_i\,) \bullet \Delta_i$ for $i \in \{1, 2, 3\}$. For A_1, $\Psi|A_1$ is specified by $\overline{a}bcd \prec_{\Psi|A_1} \overline{a}bc\overline{d}$. So, in the revised state $(\,\Psi|A_1\,)^\bullet = (\,\Psi|A_1\,) \bullet \Delta_1$, we must have $\overline{a}bc\overline{d} \prec_{(\,\Psi|A_1\,)^\bullet} \overline{a}bcd$. For A_2, we apply a qualitative c-revision and obtain for the revised state $(\,\Psi|A_2\,)^\bullet = (\,\Psi|A_2\,) \bullet \Delta_2$ the total preorder $\overline{a}\overline{b}c\dot{d} \prec_{(\,\Psi|A_2\,)^\bullet} \overline{a}\overline{b}\overline{c}\dot{d}$. Suitable queries to $\Psi \bullet \Delta$ can then be answered locally. E.g., if we query whether $\Psi \bullet \Delta$ accepts the conditionals $(c|\overline{a}\overline{b}d)$ and $(d|\overline{a}\overline{b}c)$ whose premises both entail A_2, we can look into $(\,\Psi|A_2\,)^\bullet$ and find that $(c|\overline{a}\overline{b}d)$ is accepted, while $(d|\overline{a}\overline{b}c)$ remains undecided, as before.

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