Finite Based Contraction and Expansion via Models (Extended Abstract)

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Abstract

We propose a new paradigm for Belief Change in which the new information is represented as sets of models, while the agent's body of knowledge is represented as a finite set of formulae, that is, a finite base. We define new Belief Change operations akin to traditional expansion and contraction, and we identify the rationality postulates that emerge due to the finite representability requirement. We also analyse different logics concerning compatibility with our framework.

1 Introduction

Belief Change (Alchourrón, Gärdenfors, and Makinson, 1985) studies how an agent should rationally modify its current beliefs when confronted with a piece of information. The agent should preserve most of its original beliefs, which is known as the principle of minimal change. The standard paradigm of Belief Change (Alchourrón, Gärdenfors, and Makinson, 1985), named AGM, assumes that an agent's epistemic state is represented as a set of sentences logically closed known as theories. A main issue with theories is that they are often infinite, whilst rational agents are cognitively limited in the sense that an agent is only capable of carrying a finite amount of explicit beliefs, from which its implicit beliefs are drawn.

While in classical propositional logics, every theory can be represented via a finite set of formulae, called *finite base* (Hansson, 1996, 1993), this is not the case for more expressive logics such as first-order logic (FOL) and the Description Logics (DLs) $\mathcal{EL}$ and $\mathcal{ALC}$ (Baader et al., 2017). Computationally, the finiteness requirement is also important as reasoners usually can only deal with finite sets of formulae. Thus, it is paramount that Belief Change operations guarantee that the new epistemic state is finite based (Hansson, 2017). In fact, there are logics which are not even compatible with finiteness, that is, there are logics in which the only rational outcome is not finite based (Table 1). The absence of finiteness has appeared in the DL literature as an obstacle and is sometimes called non-axiomatizable bases (Liu et al., 2011). Moreover, although the AGM rationality postulates do not depend on any specific logic, classes of Belief Change operations have been devised upon strong assumptions about the underlying logics, especially about their language, as being closed under classical negation and being compact. In the last years, efforts have been made to replace some of these assumptions with weaker conditions in order to extend the AGM paradigm. In this work, we consider the incoming information as a set of *models*, which generalises the AGM paradigm.

Our main contributions are: (1) we extend the AGM paradigm to support the finite base requirement; (2) we generalise the input to be represented as sets of models; (3) our approach abstracts unnecessary conditionals about the underlying logic, (4) we analyse the compatibility of several logics with respect to the finiteness requirement.

2 Eviction and Reception

In this work, we view a logic as a satisfaction system (Aiguier et al., 2018) $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$, where $\mathcal{L}$ is a language, $\mathfrak{M}$ is the set of models, also called interpretations, used to give meaning to the sentences in $\mathcal{L}$, and $\models$ is a satisfaction relation which indicates that a model M satisfies a base $\mathcal{B}$ (in symbols, $M \models \mathcal{B}$).

As we are concerned about finite representability, we will focus on sets of models that have a corresponding finite base in Λ. The collection of all *finitely representable sets of models in* Λ is given by: $\mathrm{FR}(\Lambda) := \{\mathbb{M} \subseteq \mathfrak{M} \mid \exists \mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L}) : \mathrm{Mod}(\mathcal{B}) = \mathbb{M}\}$, where $\mathcal{P}_{\mathrm{f}}(\mathcal{L})$ is the set of all finite bases in $\mathcal{L}$ and $\mathrm{Mod}_{\Lambda}(\mathcal{B})$ is the set of all models in $\mathfrak{M}$ that satisfy $\mathcal{B}$ (we omit the subscript Λ when clear from the context).

Next, we define eviction (a counterpart for contraction) and reception (a counterpart for expansion).

2.1 Eviction

Eviction changes the current finite base $\mathcal{B}$ removing any interpretation in the input set $\mathbb{M}$. If $\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$ is not finitely representable, then we could simply remove more models until we obtain finite representability. Since we want to preserve as many models of the original base as possible, we look at the $\subseteq$-maximal finitely representable subsets of $\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$ (Definition 1). These sets are the closest we can get to the ideal result while keeping finite representability when subtracting $\mathbb{M}$. Before we present the eviction functions based on this idea, we introduce some auxiliary tools.

Definition 1. Let $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ be a satisfaction system and $\mathbb{M} \subseteq \mathfrak{M}$. $\mathrm{MaxFRSubs}(\mathbb{M}, \Lambda) := \{\mathbb{M}' \in \mathrm{FR}(\Lambda) \mid \mathbb{M}' \subseteq \mathbb{M}$ and $\nexists \mathbb{M}'' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M}' \subset \mathbb{M}'' \subseteq \mathbb{M}\}$.

Given a satisfaction system $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ and a set of models $\mathbb{M} \subseteq \mathfrak{M}$, the set $\mathrm{MaxFRSubs}(\mathbb{M}, \Lambda)$ contains exactly all the largest (w.r.t. $\subseteq$) finitely representable subsets of $\mathbb{M}$. Then, to contract a set $\mathbb{M}$ from a finite base $\mathcal{B}$, we can choose a finite base for one of the sets in $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M})$. However, this is only generally possible iff $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) \neq \emptyset$ for all $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ and $\mathbb{M} \subseteq \mathfrak{M}$. If this condition holds we say that Λ is *eviction-compatible*.

Definition 2. A FR selection function on a satisfaction system Λ is a map $\mathrm{sel} : \mathcal{P}^*(\mathrm{FR}(\Lambda)) \to \mathrm{FR}(\Lambda)$ such that $\mathrm{sel}(X) \in X$.

Thus, each FR selection function determines an eviction function as follows.

Definition 3. Let Λ be an eviction-compatible satisfaction system and sel a FR selection function on Λ. The *maxichoice eviction function on* Λ defined by sel is a map $\mathrm{evc}_{\mathrm{sel}} : \mathcal{P}_{\mathrm{f}}(\mathcal{L}) \times \mathcal{P}(\mathfrak{M}) \to \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ such that:
$\mathrm{Mod}(\mathrm{evc}_{\mathrm{sel}}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda))$.

The operation $\mathrm{evc}_{\mathrm{sel}}$ chooses exactly one set in $\mathrm{MaxFRSubs}$. Another approach is to allow the selection function to choose multiple elements, and then intersect all of them to build the eviction result. However, we have shown that this strategy cannot be applied in our setting.

Theorem 4. *A model change operation* evc*, defined on an eviction-compatible satisfaction system* Λ*, is a maxichoice eviction function iff it satisfies the following postulates:*

(success) $\mathbb{M} \cap \mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \emptyset$.

(inclusion) $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subseteq \mathrm{Mod}(\mathcal{B})$.

(finite retainment) *If* $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subset \mathbb{M}' \subseteq \mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}$ *then* $\mathbb{M}' \notin \mathrm{FR}(\Lambda)$.

(uniformity) *If* $\mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) = \mathrm{MaxFRSubs}(\mathrm{Mod}(\mathcal{B}') \setminus \mathbb{M}', \Lambda)$ *then* $\mathrm{Mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{Mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}'))$.

2.2 Reception

Reception alters a finite base $\mathcal{B}$ to incorporate all models in $\mathbb{M}$. In some satisfaction systems, $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}$ is not finitely representable. With an analogous strategy as the one for eviction, we define reception using the smallest supersets of $\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}$. The constructions and results for reception are similar to eviction's.

Definition 5. Let $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ be a satisfaction system and $\mathbb{M} \subseteq \mathfrak{M}$. $\mathrm{MinFRSups}(\mathbb{M}, \Lambda) := \{\mathbb{M}' \in \mathrm{FR}(\Lambda) \mid \mathbb{M} \subseteq \mathbb{M}'$ and $\nexists \mathbb{M}'' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M} \subseteq \mathbb{M}'' \subset \mathbb{M}'\}$.

Similarly, we can define reception in Λ iff Λ is *reception-compatible*. That is, if and only if $\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda) \neq \emptyset$ for all $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ and $\mathbb{M} \subseteq \mathfrak{M}$.

Definition 6. Let $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ be a reception-compatible satisfaction system and sel a FR selection function on Λ. The *maxichoice model reception function on* Λ defined by sel is a map $\mathrm{rcp}_{\mathrm{sel}} : \mathcal{P}_{\mathrm{f}}(\mathcal{L}) \times \mathcal{P}(\mathfrak{M}) \to \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ such that:
$\mathrm{Mod}(\mathrm{rcp}_{\mathrm{sel}}(\mathcal{B}, \mathbb{M})) = \mathrm{sel}(\mathrm{MinFRSups}(\mathrm{Mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda))$.

We also identify the set of rationality postulates that characterise the reception function from Definition 6. These postulates are analogous to those for eviction.

3 Compatibility

We analyse some satisfaction systems and establish whether they are (or not) eviction- and reception-compatible. When $\mathrm{FR}(\Lambda)$ is finite, eviction- and reception-compatibility depend only on properties of the lattice formed by $\mathrm{FR}(\Lambda)$ under $\subset$. Theorem 7 is a consequence of this.

Theorem 7. *Let* $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ *be satisfaction system with* $\mathrm{FR}(\Lambda)$ *finite. Then (1)* Λ *is eviction-compatible iff* $\emptyset \in \mathrm{FR}(\Lambda)$ *and (2)* Λ *is reception-compatible iff* $\mathfrak{M} \in \mathrm{FR}(\Lambda)$.

By Theorem 7, we can see that the framework we presented in the previous section is general enough to cover several satisfaction systems without imposing much constraints upon the logics being used to represent an agent's beliefs. In particular, it covers propositional logic and Horn logic (containing $\bot$) since in this case $\mathrm{FR}(\Lambda)$ is finite and we can represent both $\emptyset$ and $\mathfrak{M}$.

Other interesting cases that are also both eviction- and reception-compatible are Kleene's 3-valued (K3) logics (Kleene, 1952) and Gödel propositional logic (Hájek, 1998; Bergmann, 2008). The latter is one of the most important fuzzy logics and, in contrast with the previous cases, there are infinitely many models. Priest's 3-valued (P3) logics (Priest, 1979) cannot represent $\emptyset$ and, therefore, it is not eviction-compatible. There are well-known fragments of first-order logic used for knowledge representation that are neither eviction nor reception-compatible, as it is the case of the classical DL $\mathcal{ALC}$. Other popular, DLs, such as DL-Lite$_{\mathcal{R}}$, have characteristics similar to propositional logic (assuming a finite signature) and fit with the eviction- and reception-compatibility notions, even though models can be infinite. Table 1 summarises eviction- and reception-compatibility for various satisfaction systems.

Satisfaction System	Compatible	
	Eviction	Reception
Λ(Prop), Λ(Horn)	Yes	Yes
Λ(K3), Λ(Gödel, θ)	Yes	Yes
Λ(P3), Λ(LTL$_{\mathrm{X}}$)	No	Yes
Λ(DL-Lite$_{\mathcal{R}}$)†	Yes	Yes
$\Lambda(\mathcal{ALC})$	No	No

Table 1: Eviction- and reception-compatibility of different satisfaction systems. †: only with finite signature

4 Conclusion

We introduced a new paradigm of Belief Change: the incoming information is represented as a set of models and the agent's epistemic state is represented as a finite base. The agent can either incorporate the incoming models (via reception) or remove them (via eviction). In either case, the resulting belief base must be finitely representable. We leave model revision as a future work. We envisage that the results we obtain for eviction and reception shall shed light towards this other operation. Another line of research concerns the effects of partially constraining the structure of the resulting base, in the spirit of pseudo-contractions.

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