

ASP in Industry, here and there

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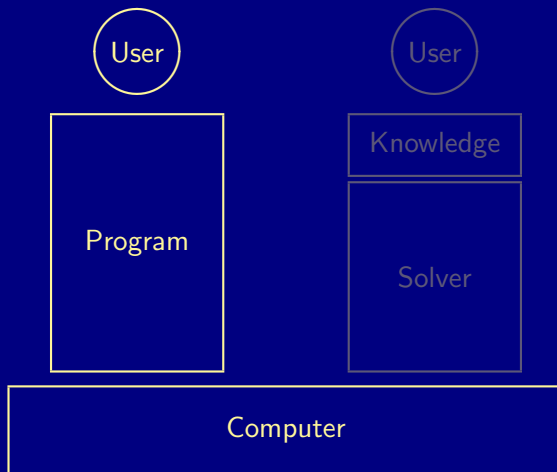
Outline

- 1 Motivation
- 2 Nutshell
- 3 Foundation
- 4 Usage
- 5 At work
- 6 Omissions
- 7 Recap

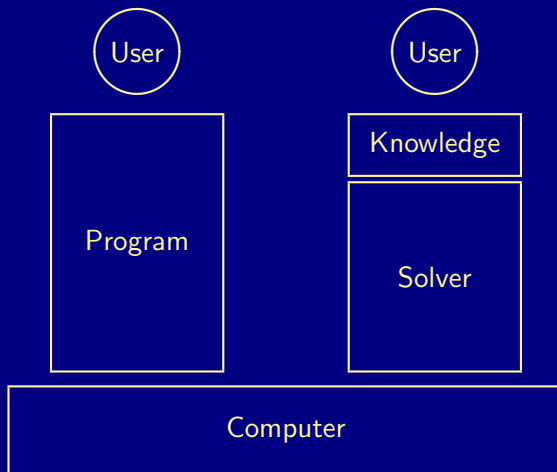
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Traditional Software



Knowledge-driven Software



What is the benefit?

- + Transparency
- + Flexibility
- + Maintainability
- + Reliability

- + Generality
- + Efficiency
- + Optimality
- + Availability

Knowledge

Expert

Solver

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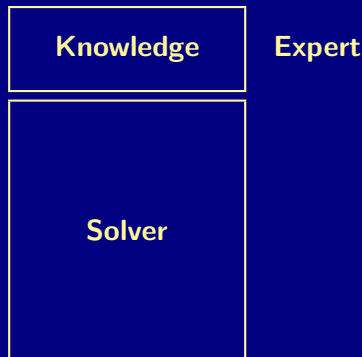
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Industrial impact

Within SIEMENS, constraint technologies have been successfully used for solving configuration problems for more than 25 years. [...] approximately 80 percent of the maintenance costs and more than 60 percent of the development costs for the knowledge representation and reasoning tasks were saved.

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ASP is an approach for declarative problem solving

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- Where is ASP from?

- Databases
- Logic programming
- Knowledge representation and reasoning
- Satisfiability solving

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Examples Sudoku, Configuration, Diagnosis, Music composition, Planning, System design, Time tabling, etc.

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- Debian, Ubuntu: Linux package configuration
- Exeura: Call routing
- FCC: Radio frequency auction
- Gioia Tauro: Workforce management
- NASA: Decision support for Space Shuttle
- SBB: Train disposition
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Over 13 months in 2016–17 the **US Federal Communications Commission** conducted an “incentive auction” to repurpose radio spectrum from broadcast television to wireless internet. In the end, the auction yielded **\$19.8 billion**, \$10.05 billion of which was paid to 175 broadcasters for voluntarily relinquishing their licenses across 14 UHF channels. Stations that continued broadcasting were assigned potentially new channels to fit as densely as possible into the channels that remained. The government netted more than **\$7 billion** (used to pay down the national debt) after covering costs. A crucial element of the auction design was the construction of a **solver**, dubbed SATFC, **that determined whether sets of stations could be “repacked” in this way; it needed to run every time a station was given a price quote.** This

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- What are ASP's distinguishing features?

- High level, versatile modeling language
- High performance solvers
- Qualitative and quantitative optimization

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- What are ASP's distinguishing features?

- High level, versatile modeling language
- High performance solvers
- Qualitative and quantitative optimization

- Any industrial impact?

- ASP Tech companies: DLV Systems and **Potassco Solutions**
- Increasing interest in (large) companies

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Open and Closed world reasoning

■ Open world reasoning

- if a statement is true, it remains true
- if a statement is false, it remains false
- if a statement is unknown, it is either true or false

■ Closed world reasoning

- if a statement is true, it remains true
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- ASP offers both open and closed world reasoning by using stable model semantics

Logic programs

- A logic program, P , over a set \mathcal{A} of atoms is a finite set of rules
- A rule is of the form

$$a_0 \text{ :- } a_1, \dots, a_m, \text{ not } a_{m+1}, \dots, \text{ not } a_n.$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

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- Semantics given by stable models, informally, models of P justifying each true atom by a proof

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Minimal models in the logic HT (Heyting'30) / G3 (Gödel'32)

Open and Closed world reasoning

by example

- Alphabet $\{a, b\}$

- The rule

- a

has the

- models $\{a\}, \{a, b\}$

- minimal models $\{a\}$

- stable models $\{a\}$

Open and Closed world reasoning

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- Alphabet $\{a, b\}$

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has the

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Open and Closed world reasoning

by example

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- The rule

- $\neg b \rightarrow a$

has the

- models $\{a\}, \{b\}, \{a, b\}$
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The logic of Here-and-There (HT)

- Formula $\varphi ::= \perp \mid a \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi$
- Interpretation A pair $\langle H, T \rangle$ of sets of atoms with $H \subseteq T$
 - H is called “here” and
 - T is called “there”
- Note $\langle H, T \rangle$ is a simplified Kripke structure
- Intuition
 - H represents provably true atoms
 - T represents possibly true atoms
 - atoms not in T are false
- Idea
 - $\langle H, T \rangle \models \varphi \sim \varphi$ is provably true
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Satisfaction

- $\langle H, T \rangle \models a$ if $a \in H$ for any atom a
- $\langle H, T \rangle \models \varphi \wedge \psi$ if $\langle H, T \rangle \models \varphi$ and $\langle H, T \rangle \models \psi$
- $\langle H, T \rangle \models \varphi \vee \psi$ if $\langle H, T \rangle \models \varphi$ or $\langle H, T \rangle \models \psi$
- $\langle H, T \rangle \models \varphi \rightarrow \psi$ if $\langle X, T \rangle \models \varphi$ implies $\langle X, T \rangle \models \psi$
for both $X = H, T$
- Note $\langle H, T \rangle \models \neg\varphi$ if $\langle T, T \rangle \not\models \varphi$ since $\neg\varphi = \varphi \rightarrow \perp$
- An interpretation $\langle H, T \rangle$ is a model of φ , if $\langle H, T \rangle \models \varphi$

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Tautologies

H	T	a	$\neg a$	$a \vee \neg a$	$\neg \neg a$	$\neg \neg a \vee \neg a$	$a \leftrightarrow \neg \neg a$
$\{a\}$	$\{a\}$	T	F	T	T	T	T
\emptyset	$\{a\}$	F	F	F	T	T	F
\emptyset	\emptyset	F	T	T	F	T	T

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\emptyset	$\{a\}$	<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>
\emptyset	\emptyset	<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>

Equilibrium models

(Pearce'96)

- A total interpretation $\langle T, T \rangle$ is an **equilibrium model** of a formula φ , if
 - 1 $\langle T, T \rangle \models \varphi$
 - 2 $\langle H, T \rangle \not\models \varphi$ for all $H \subset T$
- T is called a stable model of φ
- Note
 - $\langle T, T \rangle$ acts as a classical model
 - $\langle H, T \rangle \models P$ iff $H \models P^T$ (P^T is the reduct of P by T)

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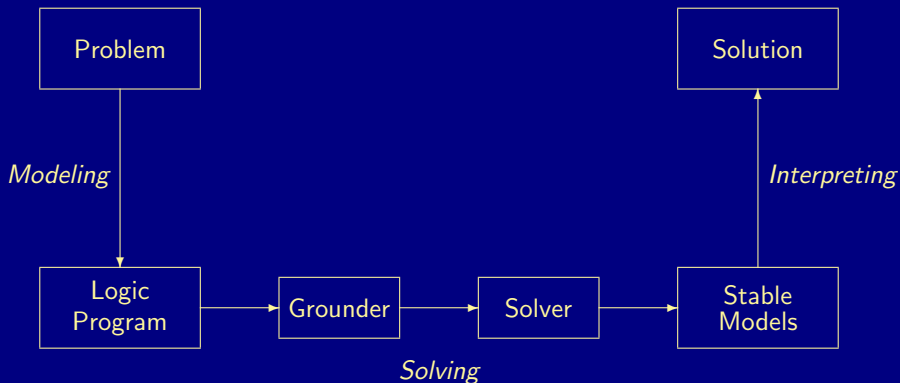
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Modeling, grounding, and solving



Language constructs

- Facts `q(42).`
- Rules `p(X) :- q(X), not r(X).`
- Conditional literals `p :- q(X) : r(X).`
- Disjunction `p(X) ; q(X) :- r(X).`
- Integrity constraints `:- q(X), p(X).`
- Choice `2 { p(X,Y) : q(X) } 7 :- r(Y).`
- Aggregates `s(Y) :- r(Y), 2 #sum{ X : p(X,Y), q(X) } 7.`
- Multi-objective optimization `:~ q(X), p(X,C). [C]`
`#minimize { C : q(X), p(X,C) }`

The traveling salesperson problem (TSP)

- Problem Instance A set of cities and distances among them, or simply a weighted graph
- Problem Class What is the shortest possible route visiting each city once and returning to the city of origin?
- Note
 - TSP extends the Hamiltonian cycle problem:
Is there a cycle in a graph visiting each node exactly once
 - TSP is relevant to applications in logistics, planning, chip design, and the core of the vehicle routing problem

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Traveling salesperson

Problem instance, `cities.lp`

```
start(a).
```

```
city(a). city(b). city(c). city(d).
```

```
road(a,b,10). road(b,c,20). road(c,d,25). road(d,a,40).
```

```
road(b,d,30). road(d,c,25). road(c,a,35).
```

Traveling salesperson

Problem encoding, tsp.lp

```
{ travel(X,Y) } :- road(X,Y,_).  
  
visited(Y) :- travel(X,Y), start(X).  
visited(Y) :- travel(X,Y), visited(X).  
  
:- city(X), not visited(X).  
  
:- city(X), 2 { travel(X,Y) }.  
:- city(X), 2 { travel(Y,X) }.
```

Traveling salesperson

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{ travel(X,Y) } :- road(X,Y,_).  
  
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:- city(X), 2 { travel(X,Y) }.  
:- city(X), 2 { travel(Y,X) }.  
  
:~ travel(X,Y), road(X,Y,D). [D,X,Y]
```

Running salesperson

```
$ clingo tsp.lp cities.lp
clingo version 5.3.1
Reading...
Solving...
Answer: 1
start(a) [...] road(c,a,35)
travel(a,b) travel(b,d) travel(d,c) travel(c,a)
visited(b) visited(c) visited(d) visited(a)
Optimization: 100
Answer: 2
start(a) [...] road(c,a,35)
travel(a,b) travel(b,c) travel(c,d) travel(d,a)
visited(b) visited(c) visited(d) visited(a)
Optimization: 95
OPTIMUM FOUND

Models      : 2
  Optimum   : yes
Optimization : 95
Calls       : 1
Time        : 0.005s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.002s
```

Running salesperson

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clingo version 5.3.1
Reading...
Solving...
Answer: 1
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Optimization: 100
Answer: 2
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travel(a,b) travel(b,c) travel(c,d) travel(d,a)
visited(b) visited(c) visited(d) visited(a)
Optimization: 95
OPTIMUM FOUND

Models      : 2
  Optimum   : yes
Optimization : 95
Calls       : 1
Time        : 0.005s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.002s
```

Running salesperson

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clingo version 5.3.1
Reading...
Solving...
Answer: 1
start(a) [...] road(c,a,35)
travel(a,b) travel(b,d) travel(d,c) travel(c,a)
visited(b) visited(c) visited(d) visited(a)
Optimization: 100
Answer: 2
start(a) [...] road(c,a,35)
travel(a,b) travel(b,c) travel(c,d) travel(d,a)
visited(b) visited(c) visited(d) visited(a)
Optimization: 95
OPTIMUM FOUND

Models      : 2
  Optimum   : yes
Optimization : 95
Calls       : 1
Time        : 0.005s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.002s
```

Running salesperson

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clingo version 5.3.1
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Outline

- 1 Motivation
- 2 Nutshell
- 3 Foundation
- 4 Usage
- 5 At work
- 6 Omissions
- 7 Recap

Routing and scheduling

Applications

- Routing
 - multi-agent path finding
 - phylogenetic inference
 - wire routing
 - etc
- Routing and Scheduling
 - train scheduling
 - embedded system design
 - warehouse robotics
 - etc
- Techniques
 - index variables by time steps
 - view time as an order on variables

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Routing and scheduling

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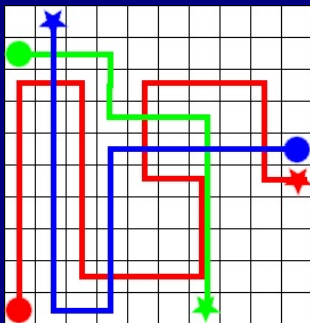
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Multi-agent path finding

- Problem Find (optimal) collision-free paths for a group of agents from their location to an (assigned) target
- Example

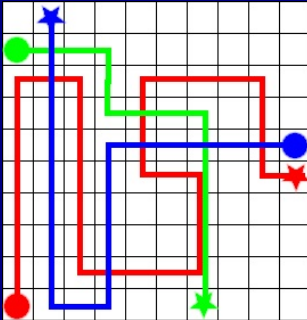
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Timestep-based routing

No scheduling yet

```
% guess moves
{ move(A,U,V,T): edge(U,V) } <= 1 :- agent(A), T=1..n.

% infer agent positions
at(A,U,0) :- start(A,U).
at(A,V,T) :- move(A,_,V,T), T=1..n.
at(A,U,T) :- at(A,U,T-1), not move(A,U,_,T), T=1..n.

% ensure path-like strolls
:- move(A,U,_,T), not at(A,U,T-1).
:- goal(A,U), not at(A,U,n).

% handle vertex/swap/follow conflicts
:- { at(A,U,T) } > 1, vertex(U), T=0..n.
:- move(_,U,V,T), move(_,V,U,T).
:- at(A,U,T), at(B,U,T+1), A!=B, m=fc.

% ensure unique agent positions (redundant/for performance)
:- { at(A,U,T) } != 1, agent(A), T=1..n.
```

Timestep-based to -free routing

in view of scheduling

- Idea $\text{move}(A, U, V, T) \rightsquigarrow \text{move}(A, U, V)$

- Pros

- no timesteps
- no explicit bound

- Cons

- no cyclic (parts of) trajectories

Timestep-free routing, part I

No scheduling yet

```
% generate moves with in and out degrees of one
{ move(A,U,V): edge(U,V) } <= 1 :- agent(A), vertex(V).
{ move(A,U,V): edge(U,V) } <= 1 :- agent(A), vertex(U).

:- move(A,U,_), not start(A,U), not move(A,_,U).
:- move(A,_,U), not goal(A,U), not move(A,U,_).

% fix in and out degrees of start and goal vertices
:- start(A,U), move(A,_,U).
:- goal(A,U), move(A,U,_).

:- start(A,U), not goal(A,U), not move(A,U,_).
:- goal(A,U), not start(A,U), not move(A,_,U).
```

Timestep-free routing, part I

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:- start(A,U), not goal(A,U), not move(A,U,_).
:- goal(A,U), not start(A,U), not move(A,_,U).
```

Timestep-free routing, part II

No scheduling yet

```
% generate order considering conflict positions
resolve(A,B,U) :- start(A,U), move(B,_,U), A!=B.
resolve(A,B,U) :- goal(B,U), move(A,_,U), A!=B.

{ resolve(A,B,U);
  resolve(B,A,U) } >= 1 :- move(A,_,U), move(B,_,U), A<B.

% discard invalid orders
:- resolve(A,B,U), resolve(B,A,U).
```

Timestep-free routing, part III

No scheduling yet

■ Acyclicity constraints

```
% check order
#edge ((A,U),(A,V)) : move(A,U,V).
#edge ((A,V),(B,U)) : resolve(A,B,U), move(A,U,V).
```

■ Difference constraints

```
% check order
&diff{(A,U)+1}<=(A,V) :- move(A,U,V).
&diff{(A,V)+1}<=(B,U) :- resolve(A,B,U), move(A,U,V).
```

■ Difference constraints

```
% check order
&diff{(A,U)+D}<=(A,V) :- move(A,U,V), edge(U,V,D).
&diff{(A,V)+D}<=(B,U) :- resolve(A,B,U), move(A,U,V), edge(U,V,D).
```


Timestep-free routing, part III

No scheduling yet

■ Acyclicity constraints (*clingo*)

```
% check order
#edge ((A,U),(A,V)) : move(A,U,V).
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```

■ Difference constraints (*clingo*[DL])

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```

Timestep-free routing and scheduling, III

■ Acyclicity constraints (*clingo*, routing only)

```
% check order
```

```
#edge ((A,U),(A,V)) : move(A,U,V).
```

```
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```

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```
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```

```
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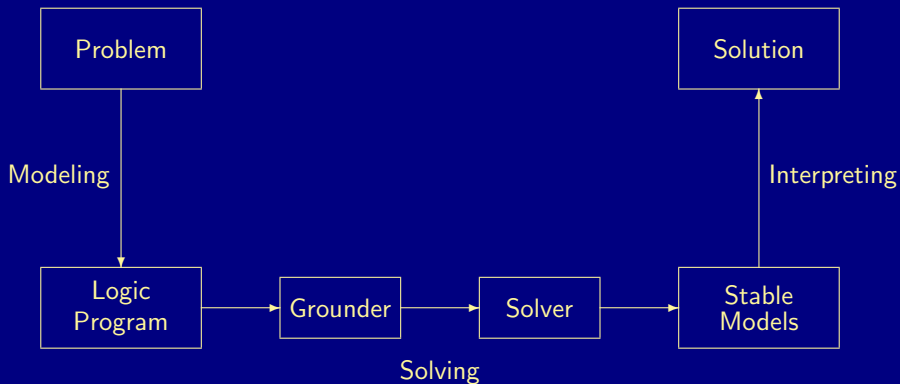
■ Difference constraints (*clingo*[DL], routing and scheduling)

```
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```

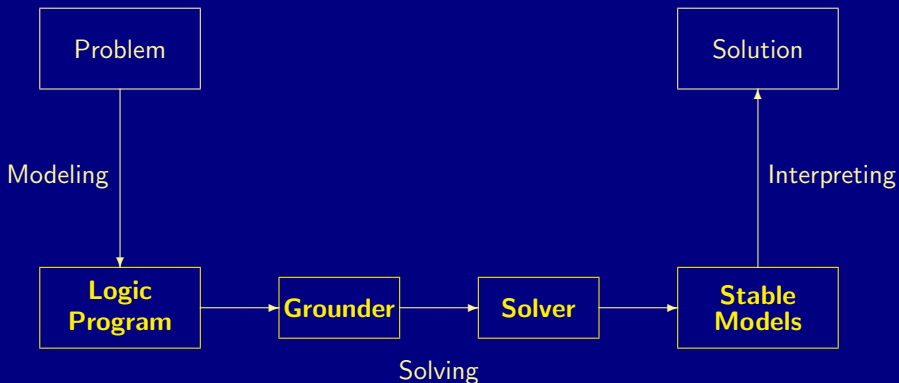
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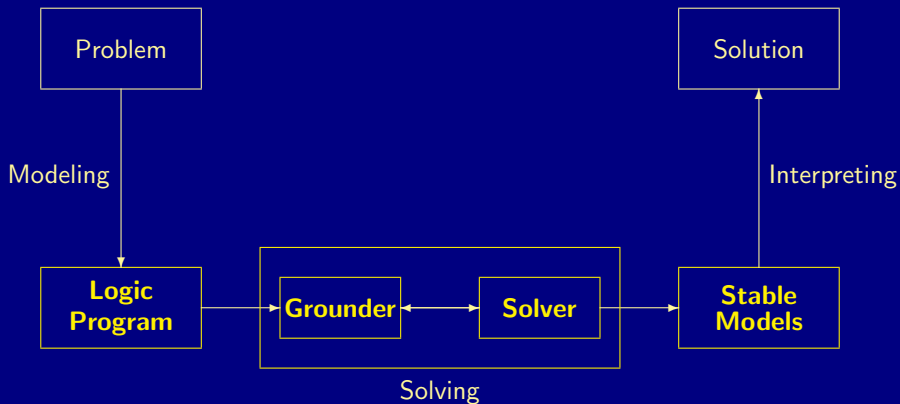
ASP solving process



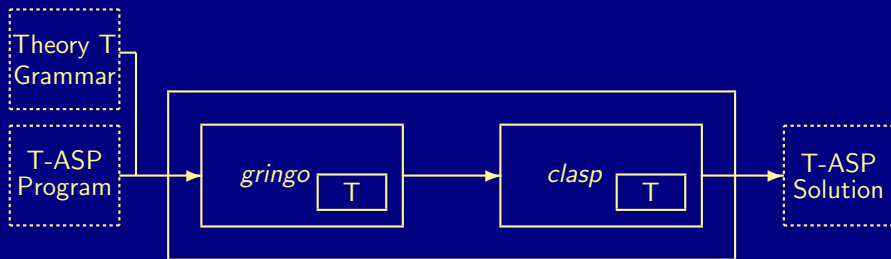
ASP solving process **modulo** theories



ASP solving process modulo theories

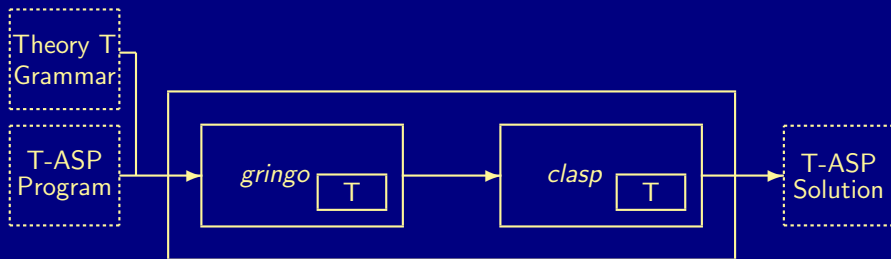


clingo's approach

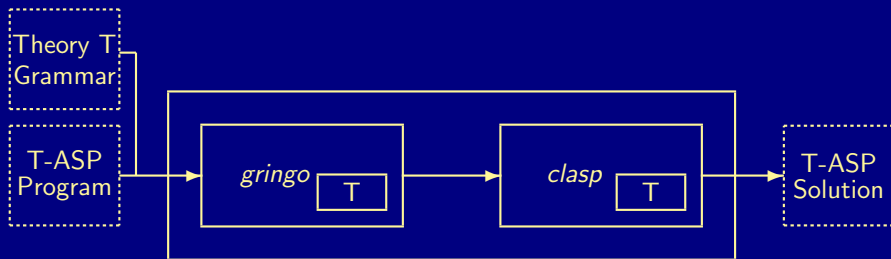


- Challenge Logic programs with elusive theory atoms
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Open and Closed world reasoning

on numeric domains

■ Open world reasoning

- if a variable occurs in true constraints, it is assigned appropriate values
- if a variable occurs in no constraint, it takes all possible values

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offers defaults, succinctness

HT_c Syntax

- Signature $\langle \mathcal{X}, \mathcal{D}, \mathcal{A} \rangle$

- \mathcal{X} variables
- \mathcal{D} domain
- \mathcal{A} atoms

- Note The syntax of atoms is left open

- Example Atom “ $x - y \leq d$ ” with $x, y \in \mathcal{X}$ and $d \in \mathcal{D}$

- HT_c-formula φ over \mathcal{A}

$$\varphi ::= \perp \mid a \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi \quad \text{where } a \in \mathcal{A}$$

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HT_c Semantics

■ Valuation $v : \mathcal{X} \rightarrow \mathcal{D} \cup \{\mathbf{u}\}$

- $\mathbf{u} \notin \mathcal{X} \cup \mathcal{D}$ stands for undefined

Set-based representation $v \subseteq \mathcal{X} \times \mathcal{D}$

- $(x, c) \in v$ and $(x, d) \in v$ implies $c = d$
- $(x, d) \notin v$ if $v(x) = \mathbf{u}$

\mathcal{V} is the set of all valuations over \mathcal{X} and \mathcal{D}

■ Atom denotation $\llbracket \cdot \rrbracket : \mathcal{A} \rightarrow 2^{\mathcal{V}}$

■ Example

$$\llbracket \text{"x - y \leq d"} \rrbracket = \{v \in \mathcal{V} \mid v(x), v(y), d \in \mathbb{Z}, v(x) - v(y) \leq d\}$$

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HT_c-satisfaction

- HT_c-interpretation over \mathcal{X}, \mathcal{D} is a pair $\langle h, t \rangle$ of valuations over \mathcal{X}, \mathcal{D} such that $h \subseteq t$
- An HT_c-interpretation $\langle h, t \rangle$ satisfies a formula φ , written $\langle h, t \rangle \models \varphi$, if the following conditions hold
 - 1 $\langle h, t \rangle \not\models \perp$
 - 2 $\langle h, t \rangle \models a$ if both $h \in \llbracket a \rrbracket$ and $t \in \llbracket a \rrbracket$ for $a \in \mathcal{A}$
 - 3 $\langle h, t \rangle \models \varphi \wedge \psi$ if $\langle h, t \rangle \models \varphi$ and $\langle h, t \rangle \models \psi$
 - 4 $\langle h, t \rangle \models \varphi \vee \psi$ if $\langle h, t \rangle \models \varphi$ or $\langle h, t \rangle \models \psi$
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- t is called an HT_C-stable model of φ

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HT_c benefits

- Semantic framework for ASP modulo theory systems (AMT) combining closed and open world reasoning
 - conservative extension of HT
 - flexibility due to open syntax and denotational semantics
 - study of AMT systems
 - study of language fragments
 - soundness of program transformations
 - warrant substitution of equivalent expressions
 - etc.

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- Heuristic programming
- Application interface programming
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Take home message

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Modeling + Grounding + Solving

Take home message

Modeling + Grounding + Solving

ASP = DB+LP+KR+SAT

Take home message

Modeling + Grounding + Solving

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And it's fun !